Math 241, Quiz 9. 10/24/11. Name: __________________

• Read problems carefully. Show all work. No notes, calculator, or text.
• There are 15 points total.

§14.8, #5 (15 points): Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x^2y$ subject to the constraint $g(x, y) = x^2 + 2y^2 = 6$. Show your work.

Solution: We compute

$$\nabla f(x, y) = (2xy, x^2), \quad g(x, y) = (2x, 4y).$$

Setting $\nabla f(x, y) = \lambda g(x, y)$, we solve the system

$$2xy = 2\lambda x, \quad x^2 = 4\lambda y, \quad g(x, y) = x^2 + 2y^2 = 6.$$

First, we have

$$2xy = 2\lambda x \implies x(y - \lambda) = 0 \implies x = 0 \text{ or } y = \lambda.$$

1. Suppose that $x = 0$. Substituting in $x^2 = 4\lambda y$ yields $\lambda y = 0$. Hence, we have $\lambda = 0$ or $y = 0$.
   
   (a) If $y = 0$, then we have $g(x, y) = g(0, 0) = 6$ since $x = 0$ also, a contradiction. So $f(x, y)$ does not have a max or min at $(0, 0)$.
   
   (b) If $\lambda = 0$, we only get more information from $g(x, y) = g(0, y) = 6$. In particular, $2y^2 = 6$ implies that $y = \pm\sqrt{3}$. Therefore, $f(x, y)$ may have a max or min at $(0, \pm\sqrt{3})$.

2. Suppose that $y = \lambda$. Substituting in $x^2 = 4\lambda y$ gives $x^2 = 4y^2$, which implies that $x = \pm 2y$. Substituting $x^2 = 4y^2$ in $g(x, y) = 6$ gives $x^2 + 2y^2 = 4y^2 + 2y^2 = 6y^2 = 6$, which holds if and only if $y = \pm 1$. Since $x = \pm 2y$, we find that $f(x, y)$ may have a max or min at the points $(\pm 2, 1)$ and $(\pm 2, -1)$.

It remains to evaluate $f(x, y) = x^2y$ at the six points where $f(x, y)$ could have a max or min. We find that

$$f(0, \pm\sqrt{3}) = 0, \quad f(\pm 2, 1) = 4, \quad f(\pm 2, -1) = -4.$$

It follows that $f(x, y)$ has a max at $(\pm 2, 1)$ and a min at $(\pm 2, -1)$. 