Math 241, Quiz 7. 10/10/11.

Name: ______________________

• Read problems carefully. Show all work. No notes, calculator, or text.
• There are 15 points total.

1. §14.5, #5 (5 points): Use the chain rule to find \( \frac{dw}{dt} \) when

\[
w = x e^{y/z}, \quad x = t^2, \quad y = 1 - t, \quad z = 1 + 2t.
\]

You do not have to convert \( x, y, z \) back to \( t \) in your final answer; your final answer can be in terms of \( x, y, z, \) and \( t \).

**Solution:** The chain rule in this setting has the form

\[
\frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt}.
\]

We compute:

\[
\frac{\partial w}{\partial x} = e^{y/z}, \quad \frac{dx}{dt} = 2t, \quad \frac{\partial w}{\partial y} = \frac{x}{z} e^{y/z}, \quad \frac{dy}{dt} = 1, \quad \frac{\partial w}{\partial z} = -\frac{xy}{z^2} e^{y/z}, \quad \frac{dz}{dt} = 2.
\]

Hence, we have

\[
\frac{dw}{dt} = e^{y/z}(2t) + \frac{x}{z} e^{y/z}(-1) - \frac{xy}{z^2} e^{y/z}(2) = e^{y/z} \left( 2t - \frac{x}{z} - \frac{2xy}{z^2} \right).
\]

2. §14.6, #11 (5 points): Find the directional derivative of \( f(x, y) = 1 + 2x \sqrt{y} \) at the point \((3, 4)\) in the direction of \( \mathbf{v} = \langle 4, -3 \rangle \).

**Solution:** We compute

\[
\nabla f(x, y) = \langle 2\sqrt{y}, 2x(1/2)y^{-1/2} \rangle = \langle 2\sqrt{y}, xy^{-1/2} \rangle \implies \nabla f(3, 4) = \langle 4, 3/2 \rangle.
\]

A unit vector in the direction of \( \mathbf{v} \) is \( \mathbf{u} \langle 4/5, -3/5 \rangle \). Hence, we have

\[
D_{\mathbf{u}} f(3, 4) = \langle 4, 3/2 \rangle \cdot \langle 4/5, -3/5 \rangle = 16/5 - 9/10 = 23/10.
\]

3. §14.6, #29 (5 points): Find all points at which the direction of fastest change of \( f(x, y) = x^2 + y^2 - 2x - 4y \) is \( \mathbf{i} + \mathbf{j} \).

**Solution:** The direction of fastest change is \( \nabla f(x, y) = (2x - 2)\mathbf{i} + (2y - 4)\mathbf{j} \). It suffices to find all points \((x, y)\) with \( \nabla f(x, y) \parallel \mathbf{i} + \mathbf{j} \). We have

\[
\nabla f(x, y) \parallel \mathbf{i} + \mathbf{j} \iff \exists k \text{ with } 2x - 2 = k, \ 2y - 4 = k
\]

\[
2x - 2 = 2y - 4 \iff x - 1 = y - 2 \iff y = x + 1.
\]

The direction of fastest change is \( \mathbf{i} + \mathbf{j} \) for all points \((x, y)\) on \( y = x + 1 \).