1. §6.3, #19 (6 points): Use the method of cylindrical shells to setup an integral which gives the volume generated by rotating the region bounded by the given curves about the specified axis. Sketch the region and a typical shell. Do not compute the integral; only the setup is required.

region: \( y = x^3, \ y = 0, \ x = 1; \)
axis: \( y = 1. \)

Solution: Draw the region and the rotation axis. Draw a typical rectangle; it should be oriented horizontally. Therefore, its thickness is \( \Delta y. \) The volume of a typical shell is

\[
2\pi rh\Delta r = 2\pi (1-y)(1-\sqrt[3]{y})\Delta y.
\]

Hence, the volume of the solid of revolution is

\[
\int_0^1 2\pi (1-y)(1-\sqrt[3]{y}) \, dy.
\]
2. §11.1, #11 (3 points): Find a formula for the general term $a_n$ of the sequence, assuming that the pattern of the first few terms continues. **No justification required.**

$$
\{2, 7, 12, 17, \ldots \}.
$$

**Solution:** $a_n = 2 + 5(n - 1) = -3 + 5n$.

3. §11.1, #27 (3 points): Determine whether the sequence **converges** or **diverges**. If it converges, find the limit. **Briefly explain.**

$$
a_n = \cos \left(\frac{n}{2}\right).
$$

**Solution:** The sequence **diverges** since it does not approach any single number as $n \to \infty$; instead, it takes on values between $-1$ and $1$.

4. §11.1, #41 (3 points): Determine whether the sequence **converges** or **diverges**. If it converges, find the limit. **Briefly explain.**

$$
a_n = \ln(2n^2 + 1) - \ln(n^2 + 1).
$$

**Solution:**

$$
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \ln \left(\frac{2n^2 + 1}{n^2 + 1}\right) = \ln \left(\lim_{n \to \infty} \frac{2n^2 + 1}{n^2 + 1}\right) = \ln \left(\lim_{n \to \infty} \frac{2 + \frac{1}{n^2}}{1 + \frac{1}{n^2}}\right) = \ln 2.
$$

I.e., the sequence **converges** to $\ln 2$. We are justified in interchanging $\lim$ and $\ln$ since $\ln$ is continuous at $2$. 
