Graphical Analysis in Polar Coordinates

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Overview

One of the most challenging aspects to polar coordinates is being able to visualize the graph of a polar function, $r = f(\theta)$. An animation showing exactly how the curve is traced out as the angle moves through its domain is even more useful than a static graph of the function.

The simplest polar plots can be created with the `plot` command — with one additional argument. An animation in polar coordinates can be created easily with the `animatecurve` command.

Related Course Material/Preparation

- Calculus Text: §10.3 and §10.4. Maple Text: §3.4.
- Know the basic conversions between rectangular and polar coordinates:

$$\begin{aligned} r &= \sqrt{x^2+y^2} \qquad & x &= r\cos(\theta) \\ \tan\theta &= y/x & y &= r\sin(\theta) \end{aligned}$$

- Remember that all angles need to be specified in radians.
- Be prepared to create some surprising plots that would be almost impossible to create in rectangular coordinates.

Maple Essentials

- To identify and see some basic polar curves, you may want to check out the *PolarCurveID* and *Basic14Polar* maplets, which are available from the course website (last column in Lab 13):

  `http://www.math.sc.edu/calclab/142L-S10/labs`

- New Maple commands introduced in this lab include:

Command	Description
`arctan( y, x )`	Two-argument version of the inverse tangent. This is essentially equivalent to `arctan(y/x)` except that the signs of `x` and `y` are used to extend the range from $(-\frac{\pi}{2}, \frac{\pi}{2})$ to $(-\pi, \pi)$; this modification makes the two-argument arctan ideal for converting from rectangular to polar coordinates.
`plot( ..., coords=polar);`	Plot a function in polar coordinates. Example: > `R :=t-> 2*cos(4*t);` > `plot(R(t), t=0..2*Pi, coords=polar);`
`animatecurve`	Animated sketch of a curve. Example: The limaçon $r = 1 + 3\sin(\theta)$ could be animated as follows: > `R :=t-> 1 + 3*sin( t );` > `animatecurve([R(t),t,t=0..2*Pi], coords=polar);` **Note:** *Ned to execute* `with( plots ):` *before using* `animatecurve`.

Activities

1. Convert the following points to polar coordinates: (2,0), (3,3), (0,2), (-2,3), (-2,-5), (0,-3), (1,$-\sqrt{3}$). **Note:** *Compare the angles obtained with* `arctan(y/x)` *and* `arctan(y,x)`*.*
2. For each of the curves below:
   - Plot the curve in polar coordinates.
   - Animate the sketching of the curve.
   - When applicable, find the range that traces the curve exactly once.
     Note: *Optional arguments to the* `animatecurve` *command include:*
     - `frames=`n *creates an animation with n frames; the default is frames=16.*
     - `numpoints=`n *instructs Maple to use n points in each frame of an animation; the default is numpoints=50.*

(1)	$r = 2 + \sin(\theta)$	(2)	$r = \cos(3\theta)$	(3)	$r = \cos(4\theta)$
(4)	$r = 2 + \sin\left(\frac{5\theta}{3}\right)$	(5)	$r = \sin(\theta) + \cos\left(\frac{\theta}{3}\right)$	(6)	$r = \sin\left(\frac{\theta}{5}\right)$
(7)	$r = 3(1 - \cos(\theta))$	(8)	$r = 1 + (\cos(\theta))^3$	(9)	$r = (\cos(\theta))^2$
(10)	$r = \ln(\theta)$	(11)	$r = \frac{\theta}{2}$	(12)	$r^2 = \cos(2\theta)$

3. The polar function $r = e^{\cos(\theta)} - 2\cos(4\theta) + \left(\sin\left(\dfrac{\theta}{4}\right)\right)^3$ is called the "butterfly curve". Plot and animate the curve to see why.

Working with Maple

```
> with(plots):
```

Activity 1:

```
> x:=-1;
> y:=3;
> r:=sqrt(x^2+y^2);
> theta:=arctan(y,x);
> theta:=evalf(%);
> [r,theta];
```

Activity 2.2:

```
> r2:=t->cos(3*t);
> plot(r2(t),t=0..2*Pi,coords=polar);
> animatecurve([r2(t),t,t=0..2*Pi],coords=polar,numpoints=100);
```

We used t from 0 to 2π and it traced the curve twice. The range that traces the curve once is hence equal to π.

Activity 3:

```
> r:=t->exp(cos(t))-2*cos(4*t)+(sin(t/4))^3;
> plot(r(t),t=0..8*Pi,coords=polar);
> animatecurve([r(t),t,t=0..4*Pi],coords=polar,numpoints=100);
```

Assignment

Complete lab activities and your lab instructor will give other assignment for each section.