

Graphical Analysis in Polar Coordinates

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Overview

One of the most challenging aspects to polar coordinates is being able to visualize the graph of a polar function, $r = f(\theta)$. An animation showing exactly how the curve is traced out as the angle moves through its domain is even more useful than a static graph of the function.

The simplest polar plots can be created with the `plot` command — with one additional argument. An animation in polar coordinates can be created easily with the `animatecurve` command.

Related Course Material/Preparation

- Calculus Text: §10.3 and §10.4. Maple Text: §3.4.
- Know the basic conversions between rectangular and polar coordinates:

$$\begin{aligned} r &= \sqrt{x^2 + y^2} & x &= r \cos(\theta) \\ \tan \theta &= y/x & y &= r \sin(\theta) \end{aligned}$$

- Remember that all angles need to be specified in radians.
- Be prepared to create some surprising plots that would be almost impossible to create in rectangular coordinates.

Maple Essentials

- To identify and see some basic polar curves, you may want to check out the *PolarCurveID* and *Basic14Polar* maplets, which are available from the course website (last column in Lab 13):

<http://www.math.sc.edu/calclab/142L-F10/labs>

- New Maple commands introduced in this lab include:

| Command | Description |
|--|---|
| <code>arctan(y, x)</code> | Two-argument version of the inverse tangent. This is essentially equivalent to <code>arctan(y/x)</code> except that the signs of <code>x</code> and <code>y</code> are used to extend the range from $(-\frac{\pi}{2}, \frac{\pi}{2})$ to $(-\pi, \pi)$; this modification makes the two-argument <code>arctan</code> ideal for converting from rectangular to polar coordinates. |
| <code>plot(..., coords=polar);</code> | Plot a function in polar coordinates. Example: <pre>> R :=t-> 2*cos(4*t); > plot(R(t), t=0..2*Pi, coords=polar);</pre> |
| <code>animatecurve</code> | Animated sketch of a curve. Example: The limaçon $r = 1 + 3 \sin(\theta)$ could be animated as follows: <pre>> R :=t-> 1 + 3*sin(t); > animatecurve([R(t),t,t=0..2*Pi], coords=polar);</pre> Note: <i>Ned to execute with(plots): before using animatecurve.</i> |

Activities

1. Convert the following points to polar coordinates: $(2,0)$, $(3,3)$, $(0,2)$, $(-2,3)$, $(-2,-5)$, $(0,-3)$, $(1,-\sqrt{3})$. **Note:** Compare the angles obtained with $\arctan(y/x)$ and $\arctan(y,x)$.

2. For each of the curves below:

- Plot the curve in polar coordinates.
- Animate the sketching of the curve.
- When applicable, find the range that traces the curve exactly once.

Note: Optional arguments to the `animatecurve` command include:

- `frames=n` creates an animation with n frames; the default is `frames=16`.
- `numpoints=n` instructs Maple to use n points in each frame of an animation; the default is `numpoints=50`.

| | | |
|--|--|---|
| (1) $r = 2 + \sin(\theta)$ | (2) $r = \cos(3\theta)$ | (3) $r = \cos(4\theta)$ |
| (4) $r = 2 + \sin\left(\frac{5\theta}{3}\right)$ | (5) $r = \sin(\theta) + \cos\left(\frac{\theta}{3}\right)$ | (6) $r = \sin\left(\frac{\theta}{5}\right)$ |
| (7) $r = 3(1 - \cos(\theta))$ | (8) $r = 1 + (\cos(\theta))^3$ | (9) $r = (\cos(\theta))^2$ |
| (10) $r = \ln(\theta)$ | (11) $r = \frac{\theta}{2}$ | (12) $r^2 = \cos(2\theta)$ |

3. The polar function $r = e^{\cos(\theta)} - 2\cos(4\theta) + \left(\sin\left(\frac{\theta}{4}\right)\right)^3$ is called the “butterfly curve”. Plot and animate the curve to see why.

Working with Maple

```
> with(plots):
```

Activity 1:

```
> x:=-1;
> y:=3;
> r:=sqrt(x^2+y^2);
> theta:=arctan(y,x);
> theta:=evalf(%);
> [r,theta];
```

Activity 2.2:

```
> r2:=t->cos(3*t);
> plot(r2(t),t=0..2*Pi,coords=polar);
> animatecurve([r2(t),t,t=0..2*Pi],coords=polar,numpoints=100);
```

We used t from 0 to 2π and it traced the curve twice. The range that traces the curve once is hence equal to π .

Activity 3:

```
> r:=t->exp(cos(t))-2*cos(4*t)+(sin(t/4))^3;
> plot(r(t),t=0..8*Pi,coords=polar);
> animatecurve([r(t),t,t=0..4*Pi],coords=polar,numpoints=100);
```

Assignment

Complete lab activities and your lab instructor will give other assignment for each section.