

Review of Calculus I

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Overview

The fundamental operations of calculus are limits, derivatives, and integrals (definite and indefinite). The goal of this week's lab is to review those basic operations using Maple.

Maple Essentials

- Maple commands introduced in this lab:

Command	Description/Example
<code>:=x-></code>	define a function of x. eg: <code>f:=x->sqrt(x);</code> <code>g:=t->2*sin(t);</code>
<code>limit</code>	compute limits or sided limits: <code>limit(f(x), x=a);</code> finds the limit, $\lim_{x \rightarrow a} f(x)$. <code>limit(f(x), x=a, right);</code> finds the sided limit from right, $\lim_{x \rightarrow a^+} f(x)$. <code>limit(f(x), x=a, left);</code> finds the sided limit from left, $\lim_{x \rightarrow a^-} f(x)$.
<code>diff</code>	compute derivatives of an expression: <code>diff(f(x), x);</code> finds the derivative $\frac{df}{dx}$ of $f(x)$ with respect to x . <code>diff(f(x), x\$n);</code> finds the nth derivative $\frac{d^n f}{dx^n}$ of $f(x)$.
<code>int</code>	compute definite and indefinite integrals: <code>int(f(x), x);</code> evaluates the indefinite integral, $\int f(x) dx$; <code>int(f(x), x=a..b);</code> evaluates the definite integral, $\int_a^b f(x) dx$.
<code>plot</code>	plot one or more functions on a specified window <code>plot(f(x), x=a..b);</code> plots the graph of $y = f(x)$ for $a < x < b$; <code>plot([f(x), g(x)], x=a..b);</code> graphs two functions in a single plot

Note that Maple's **int** command does not include any constants of integration. Whenever you evaluate an indefinite integral, do not forget to include a constant of integration (+C) in your answer.

- Basic Functions and Expressions:** Maple uses `sqrt(x)` for \sqrt{x} , `abs(x)` for $|x|$, `exp(x)` for e^x , and `x^n` for x^n . Other basic functions can be typed in as what they are but you must carefully use () to group together and match up expressions as needed. You must also type in * when a multiplication is presented. For example, you need to type in `sin(x)` instead of `sindx` for $\sin x$ and `2*x/(x+y)` instead of `2x/x+y` for $\frac{2x}{x+y}$. Finally, do not forget to type in ; at the end of a line that is to be executed.
- Palettes:** The **Expression** palette can be used instead of typing the full Maple commands for many functions and operations. The **Symbol** palette contains other symbols, including π (`Pi`) and ∞ (`infinity`). You may want to create your own Favorites palette.

Related Course Material/Preparation

Review the basic terminology and properties of limits, derivatives, and integrals.

Activities

Many problems below can be solved in Maple using different interface methods: typing in a command directly, using the right-clicking, or using palettes. Try and/or combine different methods when possible. In any case, you should make it a habit to define a function or assign an expression to a name first. You should always inspect the output to verify that the function or expression was entered correctly. You can then apply operations by the right-clicking or referring the name or the equation label (if you use commands or palettes). However, when typing in directly, you need to pay special attention to the difference between a function and an expression. To refer an **expression**, you type in only the name. To refer a **function**, you need to type in the name together with a suitable variable.

1. Use Maple to compute the derivative of the following functions:

$$(i) \quad f1(x) = x^3 \ln(x) \qquad (ii) \quad f2(x) = \frac{\arctan(x)}{\sqrt{x^2 + 1}} \qquad (iii) \quad f3(t) = \frac{1 - \ln(t)}{1 + \ln(t)}$$

2. Use Maple to compute the first and 100th derivatives of $f4(x) = x \sin(2x)$.

3. Use Maple to evaluate the following indefinite and definite integrals:

$$(i) \quad \int \frac{x+1}{(x-2)^2} dx \qquad (ii) \quad \int_{-1}^3 x e^{-x^2} dx \qquad (iii) \quad \int_0^{\pi/2} \left| \frac{1}{2} - \cos x \right| dx$$

4. Use Maple to evaluate the following limits:

$$(i) \quad \lim_{x \rightarrow 0} \frac{x^2 \arccos(x)}{(\sin(2x))^2} \qquad (ii) \quad \lim_{n \rightarrow \infty} (1 + 3/n)^{2n} \qquad (iii) \quad \lim_{t \rightarrow 1^+} \tan\left(\frac{\pi}{2}t\right)$$

5. Let

$$f(x) = \ln\left(\frac{(x^2 + 2)^2}{\sqrt{4x^2 + 1}}\right).$$

In a Maple worksheet:

- (a) Define the function and find $f(8)$, $f(t^2 + 1)$.
- (b) Find and name the first and second derivatives of $f(x)$ as df and $d2f$, respectively.
Hint: If an expression is too messy, you may want to *simplify* it first.
- (c) Plot the graph of $y = d2f(x)$ on the interval $-4 < x < 4$ and estimate the intervals on which $y = f(x)$ is concave up.
- (d) Create a single plot containing the graphs of $y = f(x)$ and $y = df(x)$ for $-4 < x < 4$ and verify that $df(x) = 0$ at each critical number of $y = f(x)$. Did you estimate correctly in (c)?
- (e) Right-click on your graph to see the context menu. Under the option **Legend**, click **Show Legend**. Now, from the same context menu, click **Edit Legend**. Enter an appropriate label for the first curve. Repeat this step for the other curve in the plot.

Assignment

Review 141 Maple labs