

New Functions from Old

Douglas Meade, Ronda Sanders, and Xian Wu
Department of Mathematics

Overview

In this lab, we will use Maple to help us to calculate and simplify combinations of functions. A maplet (**Shift**) will also be introduced to help us to practice our skills identifying basic functions that have been shifted horizontally and/or vertically.

Maple Essentials

- Important Maple commands introduced in this lab:

Command	Description	Example
<code>:=</code> <code>></code>	define a function in x	<code>f:=x->2*x^2*(1-x^2);</code>
<code>simplify</code>	simplify functions/expressions	<code>simplify(f(x));</code> <code>simplify(x-(x-1)^8);</code>
<code>eval</code>	evaluate functions/expressions	<code>eval(f(x),x=2);</code> <code>eval((x-h)^9,h=0);</code>

- The **Shift** maplet is available from the course website:

<http://www.math.sc.edu/calclab/141L-S14/labs/> → [Shift](#)

Related course material/Preparation

Calculus Text: §1.3. Maple Text: §2.1.

Assignment

Complete lab activities and your lab instructor will give other assignment for each section

Activities

1. Use the **Shift** maplet to practice your skills identifying basic functions that have been shifted horizontally and/or vertically.
 - (a) From the Calculus I Suggested Lab Schedule/Assignments page under Lab 2, click on [Shift](#). You will be prompted for a username and password as these maplets are protected. You should use your Blackboard username and password. (You may need to reset your password there: login to your VIP, go to the TECHNOLOGY, choose the second one from the TECHNOLOGY Menu.)
 - (b) This opens a user interface for testing your ability to recognize shifts of seven basic functions. To see the seven basic functions, click the **Show Basic 7 Functions** button.
 - (c) To test your ability to recognize shifts of these functions, click on the **Show Shifted Graph** button. Enter the formula for the displayed graph (using valid Maple syntax) in the box labeled *Answer*, then click the **Check Answer** button.

Note: If you do not get the answer correct, the graph of your equation will be displayed in red.

2. In each of the following problems, you will use the assignment operator ($:=$) together with the arrow notation ($x \rightarrow$) to define each function. Once you have done this, the problems are straightforward. Remember that you have to call a function **together with its variable** (like $f(x)$, $f(t)$, $f(2)$, or $f(\text{whatever})$). Maple won't recognize a function just by his name (like f).

Note: You can always use the Expression, Common Symbols, and/or Favorites palettes to avoid typing so much. You may also find the labels useful.

- Find and simplify formulas for $f(x) + g(x)$, $f(x) - g(x)$, $f(x)g(x)$, and $f(x)/g(x)$.
 - a. $f(x) = 2\sqrt{x-1}$, $g(x) = \sqrt{x-1}$
 - b. $f(x) = 1 + \frac{x}{x+1}$, $g(x) = 2 - \frac{1}{x}$
- Let $f(x) = x^2 + 1$. Find and simplify each of the following.
 - a. $f(5s + 2)$
 - b. $3f(1/x)$
 - c. $f(f(x))$
- Evaluate $\frac{f(x+h)-f(x)}{h}$. Simplify your answer and then let h go to 0.
 - a. $f(x) = 3x^2 - 5$
 - b. $f(x) = \frac{1}{(x+1)^2}$
- Find and simplify compositions $(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$.
 - a. $f(x) = \frac{1+x}{1-x}$, $g(x) = \frac{x}{1-x}$
 - b. $f(x) = \sqrt{2x+3}$, $g(x) = x^2 + 1$
- Find and simplify $(f \circ g \circ h)(x)$.
 - a. $f(x) = \sqrt{x-1}$, $g(x) = x^2 + 2$, $h(x) = \cos x$
 - b. $f(x) = x^2 + 1$, $g(x) = \frac{1}{x}$, $h(x) = x^3$
- Express $F(x)$ as a composition of two functions; that is, find $f(x)$ and $g(x)$ such that $F(x) = f \circ g(x)$. Use Maple to verify the composition.

Note: Do not choose the identity ($y = x$) as one of your functions.

 - a. $F(x) = \sin^2 x$
 - b. $F(x) = \frac{3}{5+\cos x}$
 - c. $F(x) = (x^2 + 1)^{10}$
- Express $F(x)$ as a composition of three functions; that is, find $f(x)$, $g(x)$, and $h(x)$ such that $F = f \circ g \circ h(x)$. Use Maple to verify the composition.

Note: Do not choose the identity ($y = x$) as one of your functions.

 - a. $F(x) = (1 + \sin(x^2))^3$
 - b. $F(x) = \sqrt{1 - x^{1/3}}$
 - c. $F(x) = \cos^4(\sqrt{x})$

Example Problems

1. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ if $f(x) = \frac{4}{3+x^2}$. Simplify your answer and then let h go to 0.

```
> f:= x -> 4 / (3+x^2);
> (f(x+h) - f(x)) / h;
> simplify(label);
> eval(label, [h = 0, x = x]);
```

Note: You should right-click over your expression and choose an action. If you choose to type the command, use **Ctrl-L** to insert a label.

2. Find and simplify $f \circ g \circ h(x)$ if $f(x) = \frac{2}{1-x^2}$, $g(x) = \sin(x)$, and $h(x) = \sqrt{x}$.

```
> f:= x -> 2 / (1-x^2);
> g:= x -> sin(x);
> h:= x -> sqrt(x);
> f(g(h(x)));
> simplify(label);
```