

New Functions from Old

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Overview

In this lab, we will use Maple to help us to calculate and simplify combinations of functions. A maplet (**Shift**) will also be introduced to help us to practice our skills identifying basic functions that have been shifted horizontally and/or vertically.

Maple Essentials

- Important Maple commands introduced in this lab:

Command	Description	Example
<code>:=</code> <code>></code>	define a function in x	<code>f:=x->2*x^2*(1-x^2);</code>
<code>simplify</code>	simplify functions/expressions	<code>simplify(f(x));</code> <code>simplify(x-(x-1)^8);</code>
<code>eval</code>	evaluate functions/expressions	<code>eval(f(x),x=2);</code> <code>eval((x-h)^9,h=0);</code>

- The **Shift** maplet is available from the course website:

<http://www.math.sc.edu/calclab/141L-S12/labs/> → [Shift](#)

Related course material/Preparation

Calculus Text: §1.3. Maple Text: §2.1.

Assignment

Complete lab activities and your lab instructor will give other assignment for each section

Activities

1. Use the **Shift** maplet to practice your skills identifying basic functions that have been shifted horizontally and/or vertically.
 - (a) From the Calculus I Suggested Lab Schedule/Assignments page under Lab 2, click on [Shift](#). You will be prompted for a username and password as these maplets are protected. You should use your Blackboard username and password. (You may need to reset your password there: login to your VIP, go to the TECHNOLOGY, choose the second one from the TECHNOLOGY Menu.)
 - (b) This opens a user interface for testing your ability to recognize shifts of seven basic functions. To see the seven basic functions, click the **Show Basic 7 Functions** button.
 - (c) To test your ability to recognize shifts of these functions, click on the **Show Shifted Graph** button. Enter the formula for the displayed graph (using valid Maple syntax) in the box labeled *Answer*, then click the **Check Answer** button.

Note: If you do not get the answer correct, the graph of your equation will be displayed in red.

2. In each of the following problems, you will use the assignment operator ($:=$) together with the arrow notation ($x \rightarrow$) to define each function. Once you have done this, the problems are straightforward. Remember that you have to call a function **together with its variable** (like $f(x)$, $f(t)$, $f(2)$, or $f(\text{whatever})$). Maple won't recognize a function just by his name (like f).

Note: You can always use the Expression, Common Symbols, and/or Favorites palettes to avoid typing so much. You may also find the labels useful.

- Find and simplify formulas for $f(x) + g(x)$, $f(x) - g(x)$, $f(x)g(x)$, and $f(x)/g(x)$.
 - a. $f(x) = 2\sqrt{x-1}$, $g(x) = \sqrt{x-1}$
 - b. $f(x) = 1 + \frac{x}{x+1}$, $g(x) = 2 - \frac{1}{x}$
- Let $f(x) = x^2 + 1$. Find and simplify each of the following.
 - a. $f(5s + 2)$
 - b. $3f(1/x)$
 - c. $f(f(x))$
- Evaluate $\frac{f(x+h)-f(x)}{h}$. Simplify your answer and then let h go to 0.
 - a. $f(x) = 3x^2 - 5$
 - b. $f(x) = \frac{1}{(x+1)^2}$
- Find and simplify compositions $(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$.
 - a. $f(x) = \frac{1+x}{1-x}$, $g(x) = \frac{x}{1-x}$
 - b. $f(x) = \sqrt{2x+3}$, $g(x) = x^2 + 1$
- Find and simplify $(f \circ g \circ h)(x)$.
 - a. $f(x) = \sqrt{x-1}$, $g(x) = x^2 + 2$, $h(x) = \cos x$
 - b. $f(x) = x^2 + 1$, $g(x) = \frac{1}{x}$, $h(x) = x^3$
- Express $F(x)$ as a composition of two functions; that is, find $f(x)$ and $g(x)$ such that $F(x) = f \circ g(x)$. Use Maple to verify the composition.

Note: Do not choose the identity ($y = x$) as one of your functions.

 - a. $F(x) = \sin^2 x$
 - b. $F(x) = \frac{3}{5+\cos x}$
 - c. $F(x) = (x^2 + 1)^{10}$
- Express $F(x)$ as a composition of three functions; that is, find $f(x)$, $g(x)$, and $h(x)$ such that $F = f \circ g \circ h(x)$. Use Maple to verify the composition.

Note: Do not choose the identity ($y = x$) as one of your functions.

 - a. $F(x) = (1 + \sin(x^2))^3$
 - b. $F(x) = \sqrt{1 - x^{1/3}}$
 - c. $F(x) = \cos^4(\sqrt{x})$

Example Problems

1. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ if $f(x) = \frac{4}{3+x^2}$. Simplify your answer and then let h go to 0.

```
> f:= x -> 4 / (3+x^2);
> (f(x+h) - f(x)) / h;
> simplify(label);
> eval(label, [h = 0, x = x]);
```

Note: You should right-click over your expression and choose an action. If you choose to type the command, use **Ctrl-L** to insert a label.

2. Find and simplify $f \circ g \circ h(x)$ if $f(x) = \frac{2}{1-x^2}$, $g(x) = \sin(x)$, and $h(x) = \sqrt{x}$.

```
> f:= x -> 2 / (1-x^2);
> g:= x -> sin(x);
> h:= x -> sqrt(x);
> f(g(h(x)));
> simplify(label);
```