Differentiation and Tangent Lines

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Overview
We will learn in this lab how to use Maple to find derivatives and the equation of the tangent line to a curve at a given point.

Maple Essentials

• Important Maple commands introduced in this lab are:

<table>
<thead>
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<th>Command</th>
<th>Description</th>
<th>Example</th>
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<tbody>
<tr>
<td>diff(f(x),x);</td>
<td>find derivative of f(x)</td>
<td>diff(k^3<em>x^2,x);diff(k^3</em>x^2,k);</td>
</tr>
<tr>
<td>diff(f(x),x$4);</td>
<td>find nth derivative of f(x)</td>
<td>diff(x^8,x$4);diff(x*cos(x),x$100);</td>
</tr>
</tbody>
</table>

• The Tangents tutor is started from the Maple 11 interface under the tools menu:
  – Tools → Tutors → Calculus - Single Variable → Tangents ...

• The TangentLine maplet is started from the course website:
  – www.math.sc.edu/calclab/141L-F07/labs/ → TangentLine

Related course material/Preparation
§3.1 and §3.2 of the textbook (Anton, 8th edition). Recall the point-slope form of the equation of the line:

\[ y - y_1 = m(x - x_1), \]

where \((x_1, y_1)\) is a point on the line and \(m\) is the slope of the line. Next, since point \((x_1, f(x_1))\) is on the tangent line, we can substitute \(y_1 = f(x_1)\) and move it to the other side. We hence get:

\[ y = m(x - x_1) + f(x_1). \]

Finally, we know that the derivative evaluated at \(x_1\) is the same as the slope of the tangent line to the graph of \(y = f(x)\) at \(x_1\). Thus we get the following formula for the equation of the tangent line to the graph of \(y = f(x)\) at \(x_1\):

\[ y = f'(x_1)(x - x_1) + f(x_1). \]

Activities

A) We already knew how to find the derivative from the definition, that is, use Maple to find the limit

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}. \]

A more direct way is to use the command diff. To compute the derivative of \(f(x)\) with respect to \(x\), you simply type diff(f(x),x); (or use the right-clicking). It can also be used to find higher order derivatives. For example, to find the third derivative of \(x^4\), you simply type diff(x^4,x$3);. Try the following sets of examples (please pay attention to subtle differences):

1. diff(k*x^4,x); diff(k*x^4,k); diff(k*x^4,x$4); diff(k*x^4,x$5);
2. f:=x->x^4; diff(f,x); diff(f(x),x); diff(f(sin(x)),x);
3. diff(sin(x)*cos(x),x$100); diff(x^x*cos(x),x$8);

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B) Find the equation of the tangent line to the graph of \( f(x) \) at the point \((x_1, f(x_1))\) for the following functions. Graph the function and its tangent line on the same axes.

1. \( f(x) = x^2, x_1 = 1 \)
2. \( f(x) = 2^x, x_1 = 1 \)
3. \( f(x) = \cos(x), x_1 = \frac{\pi}{4} \) (Recall that, in Maple, you type Pi for \( \pi \))

**Example Problem**

We will do an example together for \( f(x) = x^3 - 2x + 1 \) at \( x_1 = 2 \) in two different ways:

The first way:

1. Launch the *Tangents* tutor.
2. Enter the function as \( x^3-2x+1 \) and \( x=2 \), and change the number of iterations to 5.
3. Click **Display**. The tutor will display the function and a series of secant lines, including the tangent line. The equation of the tangent line is displayed on the right.
4. Press the **Animate** button. The tutor will show the progression through the secant lines as \( \Delta x \) gets smaller.
5. The tutor will return the last graph when you click **Close**.
6. If you want to graph the function and the tangent line, assign both in a Maple worksheet and write a plot command.

The second way:

1. Define the function and assign it to \( f \).
   
   \[
   > f := x \rightarrow x^3-2*x+1;
   \]
2. Right-click and choose **Differentiate**. Then use a label (or right-click again) to assign this new function to \( df \).
   
   \[
   > df := label;
   \]
3. Find \( f'(2) \) and assign that value to \( m \).
   
   \[
   > m := df(2);
   \]
4. Find the equation of the tangent line \( y = f'(2)(x-2) + f(2) \) and assign it as a function to \( L \).
   
   \[
   > L :=x \rightarrow m*(x-2)+f(2);
   \]
5. Plot the function and the tangent line using different linestyles.
   
   \[
   > \text{plot(} [f(x), L(x)], x=-1..3, \text{linestyle=[solid, dash]}; \]

C) The **TangentLine** maplet is a great tool to practise finding the equation of the tangent line by hand. Launch the maplet and click **New Function**. Follow the prompts step by step to find \( f(a) \), \( f'(x) \), and \( f'(a) \). Then enter the equation of the tangent line as follows:

\[
 y = f'(a)(x - a) + f(a).
\]

The maplet will check each of your answers (or on Show if you don’t know the answer) and let you know whether you are correct. Please try a few problems to make sure that you really understand how to find the equation of the tangent line by hand.

**Assignment**

Exercise 41, 42, 59, and 60 of §3.6 on pages 214-215.