Worksheet 9

1. Consider the predator-prey model with equations

$$\begin{aligned} \frac{du}{dt} &= .4u - 0.008uv \\ \frac{dv}{dt} &= -.6v + 0.01uv \end{aligned}$$

a. Describe the behavior of u in the absence of v.

b. Describe the behavior of v in the absence of u.

c. Based on the equations, which of u and v are the prey and which are the predators? Explain.

d. Find the size of the predator population that is needed in order to keep the prey population constant.

e. Find the size of the prey population that is needed in order to keep the predator population constant.

f. Find the equilibrium and draw the state space of the system. Indicate the short term behavior of the system by arrows in each region of the place. Describe what happens in the long run.

2. Consider the predator-prey model from problem **1.**.

Assume initial values $u(0) = 45$, $v(0) = 50$. Use Euler's method with $\Delta t = 1$ to obtain values for $u(n)$, $v(n)$ at time $n = 1, 2, 3, 4$, etc.

a. Plot the points $(u(n), v(n)$ on the picture of the state space from problem **1.** (f) for $n = 2, 3$, $n = 7, 8$, $n = 11, 12$, $n = 17, 18$, $n = 22, 23$.

b. Draw a plot in which you represent the values of $u(n)$ as a function of n (put n on the horizontal axis and u on the vertical axis).

c. Draw a plot in which you represent the values of $v(n)$ as a function of n (put n on the horizontal axis and v on the vertical axis).

d. What seems to be the long term behavior of the system? Describe your conclusion in words.

3. Consider the predator-prey system given by the equations

$$\begin{cases} \frac{dV}{dt} &= 4V - .06V^2 - .8VP \\ \\ \frac{dP}{dt} &= -6P + 0.1VP \end{cases}$$

a. Describe what happens to each population in the absence of the other.

b. What are the equilibrium pairs in which only one of the populations is present? Give a biological explanation of your answer.

c. Is there any equilibrium in which both populations are present? If so, find it.

d. How does the value of V in the equilibrium pair you found in part c. compare to the carrying capacity for V? What is the biological reason for this?

e. Draw the state-space with the two isoclines. For each of the four regions delimitated by the isoclines, indicate the direction in which the system evolves by placing an arrow with the correct orientation in that region.

4. Consider the predator-prey system given by the equations

$$\begin{cases} \frac{dV}{dt} & = & 4V - .1V^2 - .8VP \\ \\ \frac{dP}{dt} & = & -6P + 0.1VP \end{cases}$$

Is there any equilibrium in which both populations are present? What is the biological reason for this?