Math 172 Fall 2012 WS 7

1. Compute the equilibrium point of the discrete model system

\[
\begin{align*}
  u_n &= 3u_{n-1} - 2v_{n-1} + 4 \\
  v_n &= 5u_{n-1} - 3v_{n-1} - 28
\end{align*}
\]

2. You are studying competition between red and black desert scorpions. If only a single species is present, it would grow following the logistic model, with a carrying capacity \( K_1 = 100 \) for the red scorpion, and \( K_2 = 150 \) for the black scorpion.
   a. Write the logistic model equations for each of the species.
   b. Write the Lotka-Volterra competition model equations for the two species, assuming competition coefficients of \( \alpha = 2 \) for the red scorpion, and \( \beta = 3 \) for the black scorpion.
   c. Find the equilibrium values for the equations in b.
   d. Assume the initial populations are 25 red scorpions and 50 black scorpions. Graph the state space and the isoclines for each species. Predict the short-term dynamics of the populations, and the final outcome of the competition.

3. Repeat questions c. and d. in problem 2 with \( K_1 = 100, \ K_2 = 150, \ \alpha = 0.5, \ \beta = 2 \). Assume the initial values to be:
   a. \( N_1 = 90, \ N_2 = 120 \)
   b. \( N_1 = 60, \ N_2 = 60 \)
   c. \( N_1 = 60, \ N_2 = 20 \).

4. Repeat questions c. and d. in problem 2 with \( K_1 = 100, \ K_2 = 150, \ \alpha = 0.5, \ \beta = 0.75 \). Assume the initial values of the population to be \( N_1 = 50, \ N_2 = 70 \).