**Math 172 Worksheet 4**

**1.** An algal population is governed by the equation

$$\frac{dA}{dt} = 0.0003A(200 - A)$$

Assume that the initial value is $A(0) = 50$.

a. Rewrite this in the form of a logistic equation. Specify the numerical values for the intrinsic per capita growth rate $r$ and the carrying capacity $K$.

b. Apply stepwise estimation using Euler's method to estimate the size of the population after 12 years. Use three steps, so $\Delta t = 4$.

c. Repeat part b. but this time use 12 steps, so $\Delta t = 12$.

d. Find the equilibrium values for this population and for each equilibrium value decide if it is stable or unstable.

e. Find the value of $A$ when $A$ is increasing most rapidly.

**2.** A population is modeled by the logistic equation

$$\frac{dP}{dt} = 0.04P\left(1 - \frac{P}{80}\right)$$

I. Sketch the graph of $P(t)$ in each of the following cases:

a. $P(0) = 20$

b. $P(0) = 100$

c. $P(0) = 60$.

For each graph, explain how you decide whether the function should be increasing or decreasing, concave up or concave down.

II. Now suppose 0.04 is replaced by 0.01 in the equation. Redo the graphs from a., b., c and show how the new graphs compare to the ones from part I.

**3.** The growth of a population is governed by the equation

$$\frac{dP}{dt} = 0.04P\left(1 - \frac{P}{150}\right)\left(\frac{P}{30} - 1\right)$$

I. Find all the equilibrium values and for each equilibrium value decide whether it is stable or unstable. Predict the long term outcome for this population depending on what initial value it starts with.

II. Sketch the graph of $P = P(t)$ if $P(0) = 25$.

III. Sketch the graph of $P = P(t)$ if $P(0) = 50$.

For each graph explain how you decide if the function is increasing or decreasing, concave up or concave down.