Review for Exam 3

Exam 3 will be on Monday, April 12.
Topics: Competition and predation models.

In addition to this handout, please also review previous handouts that cover these topics.

1. Consider the following continuous model of a predator-prey system:

   \[
   \frac{dV}{dt} = 0.6V \left(1 - \frac{V}{100}\right) - 0.02VP \\
   \frac{dP}{dt} = -0.4P + 0.005VP
   \]

   a. What kind of growth does the victim population exhibit if there are no predators? What kind of long term trend is there for the predators if there are no victims?
   
   b. Find all the equilibrium values of the system. Show work.
   
   c. Draw the state-space and show arrows to show the short term behavior of the system in each of the four regions. What is the short term behavior if \( V = 50, P = 10 \)?

2. In a predator-prey continuous model system, the victim population grows with an intrinsic per-capita rate of 16% per year in the absence of the predators. The net growth rate is reduced by predation: each possible predator-victim interaction results in a loss of 0.8 victims. The predator population declines at a per capita rate of 25% per year in the absence of the victims. The net growth rate is however increased by predation: each possible predator-victim interaction increases the predator population by 0.1.

   Write the continuous model equations for this system.

3. A victim population \( V \) that growslogistically with intrinsic rate \( r \) and carrying capacity \( K \) suffers loss to a predator who exhibits a functional response of type III, that is the kill rate per predator is given by

   \[
   R(V) = \frac{kV^2}{V^2 + D^2}
   \]

   a. Give the formula for \( dV/dt \).
   
   b. Superimpose the predation loss rate curve on top of the logistic growth rate curve and interpret the equilibrium values that appear on the graph in biological terms and in terms of stability.
4. A system consisting of two populations is governed by the system of differential equations

\[
\frac{dN_1}{dt} = 0.5 N_1 \left( \frac{100 - N_1 - 2N_2}{100} \right)
\]
\[
\frac{dN_2}{dt} = 0.8 N_2 \left( \frac{150 - N_2 - 0.75N_1}{150} \right)
\]

a. What kind of growth does each population exhibit in the absence of the other population? What is the biological significance of the numbers 100 in the first equation, and 150 in the second equation? Does this system correspond to a competition model, or to a predator-prey model? Explain your answer.

b. Find the equilibrium values and draw the state space. Predict the long-term behavior of the system. If the answer depends on the initial value, show what the long-term behavior will be when different initial values are chosen.

c. Replace the number 150 in the second equation by 60 (in both places where it appears), and repeat question b.