A different method for solving affine equations

We wish to solve an affine equation of the form

$$\frac{dP}{dt} = rP - c \quad \text{(or} \quad \frac{dP}{dt} = c - rP \quad \text{or} \quad \Delta P = rP - c \quad \text{or} \quad \Delta P = c - rP)$$

First we find the equilibrium value $P_{\text{equil}} = \frac{c}{r}$. Next we introduce a new function: $Q = P - P_{\text{equil}}$. What differential or difference equation is satisfied by $Q$?

Note that $\frac{dQ}{dt} = \frac{dP}{dt} = rP - c$. In order to express the right hand side of this equation in terms of $Q$, we replace $P$ by $Q + P_{\text{equil}}$ so the equation becomes

$$\frac{dQ}{dt} = r(Q + P_{\text{equil}}) - c = rQ + r\frac{c}{r} - c = rQ$$

Thus $Q$ satisfies the equation $\frac{dQ}{dt} = rQ$; this is an exponential model equation, and the solution is $Q(t) = Q_0 e^{rt}$ which allows us to obtain $P(t) = Q(t) + P_{\text{equil}} = Q_0 e^{rt} + \frac{c}{r}$. The last step is to find the value of $Q_0$ if $P_0$ is given, and that is $Q_0 = P_0 - \frac{c}{r}$, so the final answer is

$$P(t) = (P_0 - \frac{c}{r}) e^{rt} + \frac{c}{r}$$

Note: If we want to solve the equation $\frac{dP}{dt} = c - rP$ instead, then the equation for $Q$ will be $\frac{dQ}{dt} = -rQ$, so we will get $Q(t) = Q_0 e^{-rt}$, and the final answer will be

$$P(t) = (P_0 - \frac{c}{r}) e^{-rt} + \frac{c}{r}$$

If we solve a difference equation $\Delta P = rP - c$ or $\Delta P = c - rP$ using this procedure, then $e^{rt}$ should be replaced by $(1 + r)^t$ and $e^{-rt}$ should be replaced by $(1 - r)^t$. 
