Additional Problems to study for Exam 1

Note: You should also study all the problems assigned so far for homework.

1. Consider the affine equation \( \frac{dP}{dt} = 0.04P - 10 \).
   a. Fill in the blanks so that the resulting statement is an accurate description of the population modeled by this equation:
      “the population increases/decreases (choose one) at an intrinsic per capita rate of ____% and there is emigration/immigration (choose one) of ____ individuals per unit of time”
   b. Predict the long term outcome for this population if \( P_0 = 50 \).
   c. Predict the long term outcome for this population if \( P_0 = 300 \).

2. Consider the affine equation \( \frac{dP}{dt} = 20 - 0.08P \).
   a. Fill in the blanks so that the resulting statement is an accurate description of the population modeled by this equation:
      “the population increases/decreases (choose one) at an intrinsic per capita rate of ____% and there is emigration/immigration (choose one) of ____ individuals per unit of time”
   b. Predict the long term outcome for this population if \( P_0 = 50 \).
   c. Predict the long term outcome for this population if \( P_0 = 300 \).

3. In this problem you will be asked to model a population with exponential growth with variable per capita growth rate; we make the assumption that the per capita growth rate will increase when the quality of the habitat improves.

   Which of the following equations models the growth of a population that has exponential growth with variable per capita growth rate and an improving habitat?
   i. \( \frac{dP}{dt} = 8t - 4 \)  
   ii. \( \frac{dP}{dt} = (8t - 4)P \)  
   iii. \( \frac{dP}{dt} = 8P - 4 \)
   iv. \( \frac{dP}{dt} = 8 \)  
   v. \( \frac{dP}{dt} = 8P \).

4. a. Write a possible equation for a population whose growth is modeled by a logistic equation with carrying capacity of 800 individuals.
   b. Using the equation from part a. assume that \( P_0 = 150 \). What is the size of the population when it is increasing most rapidly?
   c. Write a possible equation for a population whose growth is modeled by a logistic equation with Allee effect. Assume that the carrying capacity is 800 individuals and that at least 100 individuals are required in order for the population to survive.
d. Using the equation from part b., assume that $P_0 = 150$. What is the size of the population when it is increasing most rapidly?

5. a. Write a equation $\frac{dP}{dt} =$________ that has a stable equilibrium value at $P_{equil} = 50$.
   a. Write a equation $\frac{dP}{dt} =$________ that has an unstable equilibrium value at $P_{equil} = 50$. 