Additional Problems to study for Exam 1

Note: You should also study all the problems assigned so far for homework.

1. Consider the affine equation $\frac{dP}{dt} = 0.04P - 10$. a. Fill in the blanks so that the resulting statement is an accurate description of the population modeled by this equation:

"the population **increases** at an instrinsic per capita rate of **4** % (per unit of time) and there is **emigration** of **10** individuals per unit of time"

b. long term outcome for this population if $P_0 = 50$: **extinction**

c. long term outcome for this population if $P_0 = 300$: **unlimited growth**

(reason: unstable equilibrium at $P = 250$)

2. Consider the affine equation $\frac{dP}{dt} = 20 - 0.08P$. a. Fill in the blanks so that the resulting statement is an accurate description of the population modeled by this equation:

"the population **decreases** at an instrinsic per capita rate of **8** % and there is **immigration** of **20** individuals per unit of time"

b. Predict the long term outcome for this population if $P_0 = 50$: **approachees equilibrium value of 250**

c. Predict the long term outcome for this population if $P_0 = 300$: **approaches equilibrium value of 250**

3. In this problem you will be asked to model a population with exponential growth with variable per capita growth rate; we make the assumption that the per capita growth rate will increase when the quality of the habitat improves.

Which of the following equations models the growth of a population that has exponential growth with variable per capita growth rate and an improving habitat?

i. $\frac{dP}{dt} = 8t - 4$ ii. $\frac{dP}{dt} = (8t - 4)P$ iii. $\frac{dP}{dt} = 8P - 4$

iv. $\frac{dP}{dt} = 8$ v. $\frac{dP}{dt} = 8P$.

Answer: ii **4.** a. Write a possible equation for a population whose growth is modeled by a logistic equation with carrying capacity of 800 individuals.

$$\frac{dP}{dt} = P\left(1 - \frac{P}{800}\right)$$

b. Using the equation from part a. assume that $P_0 = 150$. What is the size of the population when it is increasing most rapidly?

answer: 400

c. Write a possible equation for a population whose growth is modeled by a logistic equation with Allee effect. Assume that the carrying capacity is 800 individuals and that at least 100 individuals are required in order for the population to survive.

$$\frac{dP}{dt} = P\left(1 - \frac{P}{800}\right)\left(\frac{P}{100} - 1\right)$$

d. Using the equation from part b., assume that $P_0 = 150$. What is the size of the population when it is increasing most rapidly?

Answer: approximately 553

5. a. Write a equation $\frac{dP}{dt} = -P + 50$ that has a stable equilibrium value at $P_{equil} = 50$.

a. Write a equation $\frac{dP}{dt} = P - 50$ that has an unstable equilibrium value at $P_{equil} = 50$.