Math 172 Fall 2012 Exam 3

1. (15 pts) Write down the equations for a predator prey system under the following assumptions:

- In the absence of the predators, the victim population follows a logistic model with a carrying capacity of 100 (you may take any value you want for r, or write r in the equation).
- In the absence of the victims, the predator population follows an exponential decay model with a per-capita rate of $q = -.1$
- The predators have a functional response (feeding rate) of type II (you may use any numbers for the constants that appear in the formula for the functional response).

- The presence of the victim has a positive effect on the growth of the predators, in an amount proportional to the sizes of both populations.

2. (35 pts) Consider the predator-prey model given by the equations

$$\frac{dV}{dt} = .7V - 0.005VP$$
$$\frac{dP}{dt} = -.4P + 0.04VP$$

a. Find all the possible equilibrium pairs.

b. Write down the equations of the isoclines. Draw the isoclines.

c. What is the short term behavior if the initial values are $V(0) = 12, P(0) = 110$?(your answer should indicate increasing/decreasing for each of the two populations).

d. Use the Euler method with $\Delta t = 1$ and with the initial values given in part c. to find values for $V(t)$, $P(t)$ when $t = 1, 2, \ldots, 19$. Show these values in a table of values. Keep two significant digits after the decimal point in the values.

e. Plot the values found in part d. as points in the state space. Label each point to indicate to what value of t it corresponds.

f. In a separate plot, plot the values of $P(t)$ as a function of t.

3. (35 pts) Consider the predator-prey system given by the equations

$$\frac{dV}{dt} = 4V - .2V^2 - .8VP$$
$$\frac{dP}{dt} = -7P + 0.1VP$$

a. In the absence of the predators, the victim population has a logistic behavior. What is the carrying capacity?

b. What is the size of the victim population that is necessary to keept the predator population at a constant level?

c. Are there any equilibrium values with both populations present? Justify your answer.

d. Find the isoclines and draw them. Draw arrows to show the behavior of the system in each of the regions. What is the long term outcome?

e. Change the equation for dV/dt to

$$\frac{dV}{dt} = 4V - .02V^2 - .8VP$$

Find the equilibrium value with both populations present for the new system (the equation of dP/dt remains the same)

4. (15pts) A predator population has a feeding rate per predator given by the formula

$$R(V) = \frac{12V}{15 + V}$$

a. What is the maximum amount of victims that can be consumed by the predators when there is an unlimited amount of victims available (the saturation constant)?

b. What is the size of the victim population that will cause the predators to consume half of the maximal feeding rate (that is, half of the amount found in part a.)

c. Show the graph of $R(V)$ as a function of V. Indicate the features of the graph that correspond to the answers from parts a. and b.