Math 172 Fall 2012 Exam 2

1. (25 pts) A frog population has three stages: tadpoles T_n, juveniles J_n and adults A_n.

Each year, 25% of tadpoles become juveniles and 75% of tadpoles die. There are no tadpoles that remain in the same stage at the next step. Also, 70% of juveniles become adults and 30% pf juveniles die. There are no juveniles that remain in the same stage. 40% of adults survive, th rest die.

On average, each adult produces 5 tadpoles a year. The tadpoles and juveniles don't reproduce.

a. Set up the transition matrix that describes this process.

b. The initial population consists of 10 adults (no tadpoles and no juveniles). Find the population vectors at $t = 2, t = 3$ and also at $t = 25, t = 26$.

c. Find the distribution vectors at $t = 2, t = 3$ and also at $t = 25, t = 26$. Has the population reached a stable state at $t = 2$? At $t = 25$? Justify your answers.

d. What is the probability for a tadpole to survive for two consecutive years (until it becomes an adult)? Three consecutive years? Show work to justify your answers.

2. (25 pts) A population is divided into two age classes and the transition matrix A has eigenvalues $\lambda_1 = 1.8$ and $\lambda_2 = 0.75$. The corresponding eigenvectors are v_1 and v_2 and the initial population vector is $B_0 = 3v_1 + 5v_2$.

a. Express B_1 and B_2 in terms of v_1 and v_2.

b. Express B_n in terms of v_1 and v_2.

c. We are given

$$v_1 = \begin{bmatrix} 18 \\ 25 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ 15 \end{bmatrix}$$

Use this information to find the stable distribution vector that is eventually reacheed when the population reaches a stable state.

d. Describe the exponential behavior of the total population in the long run.

More questions on the other side

3. (35pts) Consider the equations

$$\frac{dN_1}{dt} = r_1 N_1 \frac{120 - N_1 - 0.5N_2}{120}$$
$$\frac{dN_2}{dt} = r_2 N_2 \frac{180 - N_2 - 0.75N_1}{180}$$

a. Find the equilibrium pairs with only one population present. What is the biologican meaning of these values?

b. Find the equilibrium pair with both populations present (if any), or state that it doesn't exist. Show work. How do the values of N_1, N_2 at this equilibrium point compare to their carrying capacities, and why?

c. Find the equations of the isoclines and draw the isoclines.

d. Consider the initial values $N_1(0) = 60, N_2(0) = 100$. Represent this point in the state space and use a arrow to indicate the short term behavior of the system. Also state what the short term behavior is in words.

e. The isoclines divide the state space into four regions. Use arrows to represent the short term behavior if the initial value is chosen in each one of the four regions.

4. (15 pts) Let

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} \qquad u = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

a. Calculate $A \cdot B$ and $B \cdot A$.

b. Is u an eigenvector for A, and if so what is the corresponding eigenvalue? Justify your answer.

c. Is u an eigenvector for B, and if so what is the corresponding eigenvalue? Justify your answer.