Math 172 Fall 2012 Exam 1

1. (10 pts) Let $P = P(t)$ where $\Delta P = -0.3P$ and $P(0) = 25$. Find $P(1), P(2), P(3)$ and the general formula for $P(t)$. What happens to the values of $P(t)$ in the long run?

2. (20 pts) A population $F(t)$ of fruit-flies depends on time t. The initial population is $F(0) = 600$ flies. The population is censused once a week. The intrinsic growth rate is 8% per week. At each census, 12 flies are removed from the population.

a. Write a difference equation that models this process.

b. Rewrite your equation as a recursive equation.

c. Find the size of the population after 4 weeks, and also after 20 weeks.

d. What happens to the size of the population in the long run? Justify your answer.

3. (15 pts) A car is currently worth \$ 20000 and its value is decreasing by 15% per year.

a. Write a difference equation (i.e. the equation for a discrete process) for the value of the car t years from now, and find the general solution.

b. How much will the car be worth in 5 years?

c. How long does it take for the value of the car to go down to \$2000? Show work.

4. (20 pts) The differential equation

$$\frac{dP}{dt} = 0.5P - 8$$

models a population of fish.

a. Find the equilibrium value and decide whether the equilibrium is stable or not.

b. Assume that the initial population is $P(0) = 20$. Use either one of the two methods discussed in class to find the formula for $P(t)$. Show work.

5. (24 pts) Consider a logistic model with equation

$$\frac{dP}{dt} = 0.1P\left(1 - \frac{P}{200}\right)$$

a. What is the biological meaning of the constants 0.1 and 200 that appear in the equation?

b. Sketch the graph of $P = P(t)$ if $P(0) = 20$.

c. Sketch the graph of $P = P(t)$ if $P(0) = 300$.

d. Given that $P(0) = 100$, use the Euler method with step size $\delta t = 3$ to estimate the size of the population at time $t = 15$. Show work.

6. (12 pts) a. Write a possible differential equation for a population whose growth is modeled by a logistic equation with Allee effect. Assume that the carrying capacity is 800 individuals and that at least 100 individuals are required in order for the population to survive.

b. A population declines at a rate of 4 individuals per year. Write a difference equation that models this process.

c. A population declines at a rate of 4% per year. Write a difference equation that models this process.

d. A population declines at a rate of 4% per year. Simulataneously, there is immigration of 10 inidividuals per year. Write a difference equation that models this process.