Algebraic number theory (Spring 2013), Homework 5

Frank Thorne, thornef@webmail.sc.edu

Due Friday, March 22

Recall that starred (*) exercises may involve background beyond what is assumed in this course.

1. (20 points) In a page or so, explain what you learned, and/or some related topics you would like to learn better, from John Voight's lecture.

2. (5+ points) As we proved, the Minkowski bound gives a lower bound for the discriminant of a number field K in terms of the degree, and also the number of complex embeddings.

   For each $n \leq 6$, write out (as a decimal) the Minkowski lower bound for the discriminant of a number field K, and look up (using the Jones-Roberts tables, or otherwise) the smallest discriminant of any number field with that degree. Compare the data. If you have any interesting observations about number fields of small discriminant, be sure to share them.

3. (5 points) Compute the class group of $\mathbb{Q}(\sqrt{-33})$.

4. (5 points) Compute the class group of $\mathbb{Q}(\sqrt{-163})$.

5. (5 points) Compute the class group of $\mathbb{Q}(\sqrt{-14})$.

6. (5 points) Compute the class group of $\mathbb{Q}(7^{1/3})$.

7. (8 points) Compute the class group of $\mathbb{Q}(\sqrt{\alpha})$, where $\alpha^3 - \alpha - 7 = 0$.

8. (8 points) Compute the class group of $\mathbb{Q}(\zeta_{11})$.

9. (*, 12 points) Prove that the class number of $\mathbb{Q}(\sqrt{-13947137572})$.

   Hints: No, the Ankeny and Chowla result doesn't apply. Use Dirichlet's class number formula, in combination with a computer and some analytic number theory. But just asking PARI or SAGE what the class number is (it's 17852) is cheating.