

Problem Set 1 – Arithmetic Geometry, Frank Thorne (thorne@math.sc.edu)

Due Friday, January 22, 2016

- (1) Let V be a projective plane smooth conic defined over a field K of characteristic not equal to 2. Assume that V has a K -rational point P . Prove that V is isomorphic to \mathbb{P}^1 over K .

Recall that this means there are inverse morphisms $\phi : V \rightarrow \mathbb{P}^1$ and $\phi^{-1} : \mathbb{P}^1 \rightarrow V$, the defining polynomials of which are defined over K . You might find it useful to review the definition of a morphism of projective varieties – you can see Silverman, Ch. I.3 among many other sources.

It is not difficult to reduce to the case where $V = V(ax^2 + by^2 - cz^2)$. You might or might not find it helpful to make this reduction.