1. (5 points) (a) Prove (as was taken for granted in lecture) that if \( f \) and \( g \) are multiplicative functions, then so is their convolution \( f \ast g \).

(b) If \( f \) is multiplicative and \( g \) is not, must \( f \ast g \) be multiplicative? Prove or find a counterexample.

2. (5 points) Prove that \( \sum n d(n)^2 n^{-s} = \zeta^4(s) \zeta(2s) \). 

3. (5 points) Prove that \( \sum n d(n^2) n^{-s} = \zeta^2(s) \zeta(2s) \). 

4. (5 points) (Trick question. Explain.) Write out the character tables for all primitive real characters to the following moduli: 14, 15, 16, 20, 22, 24, 25, 27.

5. (7 points) Write \( g(n) \) be the number of primitive (not necessarily real) characters modulo \( n \). Prove an explicit formula for \( g(n) \). No messy computations allowed.