

Homework 2, Math 701 – Frank Thorne (thorne@math.sc.edu)

Instructions: You are welcome and encouraged to collaborate, but please write up your own solutions.

Due Friday, September 22, 2017.

1. Chapter 1 of Dummit and Foote has a number of easy exercises concerning groups. If the subject matter is new or difficult to you, you might want to solve some of them to gain more familiarity with the definitions. No need to turn them in, unless you would like comments on your solutions.
2. If g and h are elements of a group G , prove that $o(gh) = o(hg)$.
3. (Recall that I write D_n for what is labeled D_{2n} in Dummit and Foote.)
Compute the orders of all the elements in D_n .

4. Writing D_n in terms of generators and relations, i.e.,

$$D_n = \langle r, s \mid r^n = s^2 = 1, rs = sr^{-1} \rangle,$$

determine all $g \in D_n$ for which D_n is generated by r and g .

5. *Not to be turned in.* Write down some elements of $\text{Sym}(n)$ and compute their cycle decompositions, until you're confident that you know what you're doing.
6. Suppose that $\phi : S_1 \rightarrow S_2$ is a bijection between two sets. Explicitly describe an isomorphism $\text{Sym}(S_1) \rightarrow \text{Sym}(S_2)$.
7. Read the definition of the quaternion group Q_8 in Chapter 1.5 of Dummit and Foote, and construct an injective representation $Q_8 \hookrightarrow \text{GL}_2(\mathbb{C})$. (See Exercise 26 of DF.)