Homework 3 - Math 580, Frank Thorne (thornef@mailbox.sc.edu)

Due Friday, September 26

(1) Write out the multiplication table for the integers modulo 11.

   If your answer is correct, each row and each column (other than the one with all zeroes) will contain every residue class modulo 11. Why do you know that this must be the case?

(2) Write out the multiplication table for the integers mod 10.

   The phenomenon of the previous problem will *not* happen again. Explain why not.

(3) Suppose that 11 divides the sum of two squares. Prove that 11 divides each of the squares.

(4) Dudley, p. 32-33: 1, 3, 4, 5, 7, 9, 13, 15, 16.

(5) Dudley, p. 32-33, *either* 2, 6, 8, 10, 12, 14 *or* 19, 20.

(6) Dudley, p. 40-41, 1, 3, 6, 7, 10, 12, 13, 14, 15, 16.

Bonus problems:

(1) Dudley, p. 40-41, 18, 20.

(2) Let p be a prime with $p \equiv 3 \pmod 4$. Suppose that p divides the sum of two squares. Prove that p divides each of the squares.

(3) Write out a multiplication table modulo m (you must determine a suitable value of m) such that the number 3 appears exactly six times in some row.

   (It is okay to write out only part of the multiplication table once you have shown the existence of the row you claim.)