

Homework 1 - Math 580, Frank Thorne (thornef@mailbox.sc.edu)

Due Tuesday, September 2

Note: For proofs in this homework, write them roughly in the style of Euclid. You do not have to follow his style exactly, and feel free to describe triangles as being congruent (which he did not), but please do not depart too much from Euclid.

Also, don't appeal to unproved propositions beyond where we are in the text.

- (1) Prove: If $a|b$ and $b|c$, then $a|c$.
- (2) Prove: If $d|a$, then $d|ca$ for any integer c .
- (3) Dudley, p. 9: 1, 3, 5, 6, 7, 10, 14; p. 19, 1, 3, 4, 7, 8, 9, 10, 12.

Bonus problems:

- (1) We discussed the jug problem in lecture. Suppose you have two jugs of water, one of which can hold 89 gallons, and the other of which can hold 74. Note that $(89, 74) = 1$. Does this mean you can measure any integer number of gallons between 0 and $89 + 74$ gallons?
- (2) You have observed that the Euclidean algorithm does not usually take too many steps. If you apply the Euclidean algorithm to two integers m and n , how many steps do you think it will take to find the GCD of m and n ? Say, what if m and n are 6-digit numbers, what is the worst possible case?

Proofs of upper bounds would be fantastic. Failing that, submit any combination of computations, guesses, conjectures, or statements like "If we can just solve this other problem, then the answer is..."