

### Practice “Examination 3” - Math 574, Frank Thorne (thorne@math.sc.edu)

There was no Exam 3, but enumerative combinatorics will be on the final. If I'd given a midterm, it would have looked something like this (although probably this is a bit longer than a midterm would have been.)

- How many bit strings of length 9 are there?
  - How many bit strings of length 9 are there, which start with a 1 and which have at least one other 1?
  - How many bit strings of length 9 are there, such that every 1 is followed immediately by a zero?
- You are in a city with rectangular blocks. You want to walk five blocks north and three blocks east, while walking north or east for every block.
  - In how many ways can you do this?
  - Suppose now you are not allowed to go east for the first block. Now in how many ways can you do this?
- How many integers between 1 and 1000 are multiples of 2 or multiples of 5?
  - (Careful!) How many integers between 1 and 1000 are multiples of 2, multiples of 5, or multiples of 10?
- You pass five graded papers back to the five students who wrote them. In how many ways can you do this, so that no student gets his or her own paper? (Explain fully.)
- How many numbers must you pick to ensure that at least three of them have the same remainder when divided by 11?
- A computer programming team has 13 members – six men and seven women.
  - In how many ways can you choose a team of five?
  - In how many ways can you choose a team of five, with two men and three women?
  - In how many ways can you choose a team of five, with at most three men?
  - Suppose that one of the men and one of the women are a divorced couple, and refuse to work together. In how many ways can you choose a team of five, with two men and three women?
- How many functions are there from the set  $X = \{1, 2, 3\}$  to the set  $Y = \{A, B, C, D, E, F, G\}$ ? How many of these are one-to-one?
- A store sells 8 kinds of balloons with at least 30 balloons of each kind. How many different combinations of 30 balloons can be chosen? What if the store has only 10 red balloons, but at least 30 of every other kind of balloon?
- How many solutions to  $x_1 + x_2 + x_3 = 10$  are there, where  $x_1, x_2, x_3$  are nonnegative integers? What if, instead,  $x_1, x_2, x_3$  are positive integers?

10. Explain why the equation

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$$

is a consequence of the binomial theorem. In addition, *either* give a proof of this identity which does not rely on the binomial theorem, *or* prove the binomial theorem.