Homework 3 - Math 574, Frank Thorne (thornef@mailbox.sc.edu)

Due Friday, February 3 at 5:00.

Core:

3.2: 1, 5, 15, 18, 25, 36.

3.2: 3, 9, 10, 12, 39, 41.

3.4: 21, 23, 31.

1. If $f(x)$ is a function, what does it mean to say that $\lim_{x \to a} f(x) = c$? Write in English as well as using formal logic (i.e. the symbols $\forall$ and $\exists$) Please don't write down the answer while looking at your notes! (It is important to know this from memory.)

2. What does it mean to say that $\lim_{x \to a} f(x) \neq c$? Your answer should start with "There exists" or "There is". Write in English as well as using formal logical notation.

3. What does it mean to say that $\lim_{x \to a} f(x)$ does not exist? Again, answer both in English and in formal notation.

4. Prove that
$$\lim_{x \to 3} x^2 = 9.$$

5. Prove that
$$\lim_{x \to 4} x + 5 \neq 7.$$

6. Prove that
$$\lim_{x \to 0} x \sin x = 0.$$

7. Prove that
$$\lim_{x \to 0} \cos(1/x)$$

   does not exist.

Additional:

3.2, 2, 17, 19.

3.3, 4, 13-15, 33-36.

3.4, 22, 24.

Bonus: (up to 4 points)

Find a variety of examples of valid and invalid arguments in nonmathematical sources such as: newspapers (op-ed columnists are great), magazines, blogs, political speeches and debates, or any other source not specifically dealing with mathematics or logic. Your answers may be a little bit subjective and open to interpretation; if so, please explain.

Formalize the arguments using truth variables and/or predicates, and explain if appropriate. I would be especially interested to see good examples of invalid arguments! (Remember that false premises don't make an argument invalid.)

Please cite your sources (informally; no formal citation or bibliograhy is asked for, just say where they came from).

Good examples to look for are: modus ponens and modus tollens; a conditional is equivalent to its contrapositive; converse or inverse error; valid or invalid negation of quantifiers.