Describe the set of all vectors in $\mathbb{R}^2$ which are orthogonal to $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$. Draw the relevant picture and explain it thoroughly.

This is the set \( \left\{ \vec{v} \in \mathbb{R}^2 : \vec{v} \cdot \begin{bmatrix} 4 \\ -1 \end{bmatrix} = 0 \right\} \)

\[ = \left\{ \vec{v} \in \mathbb{R}^2 : \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \end{bmatrix} = 0 \right\} \]

\[ = \left\{ \vec{v} \in \mathbb{R}^2 : 4x - y = 0 \right\} \]

i.e. the set of solutions to the equation $4x - y = 0$, which forms a line.

If we let $\vec{v}_0$ be any nonzero vector in this set, e.g. $\begin{bmatrix} 1 \\ y \end{bmatrix}$, then we can also describe the line as the set of all scalar multiples of $\vec{v}_0$.

The solution set is the line $y = 4x$, and it is also the set of vectors which make right angles to $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$. 

\[ \begin{bmatrix} 4 \\ -1 \end{bmatrix} \]
1.1. By

\[ \vec{w}_1 = \frac{1}{2} (\vec{u} + \vec{v}) \], because \( \vec{w}_1 \) ends at the opposite vertex (from the origin) and \( \vec{w}_1 \) is halfway there.

\[ \vec{w}_2 = \frac{1}{2} (\vec{v} - \vec{u}) \], because it is in the direction from \( \vec{u} \) to \( \vec{v} \), and half the distance.

(b) \[ \vec{w}_3 = -\vec{w}_1 = -\frac{1}{2} (\vec{u} + \vec{v}) \]

\[ \vec{w}_4 = -\vec{w}_2 = \frac{1}{2} (\vec{u} - \vec{v}) \].

By arithmetic

(c) \[ \vec{w}_1 + \vec{w}_4 = \frac{1}{2} (\vec{u} + \vec{v}) + \frac{1}{2} (\vec{u} - \vec{v}) = \vec{u} \].

Alternatively: Shifting \( \vec{w}_4 \) to overlap with \( \vec{w}_2 \), we see that \( \vec{w}_1 + \vec{w}_4 \) has the same start and end points as \( \vec{w}_1 \) so it equals \( \vec{u} \).

(d) \( \vec{w}_3 = -\vec{w}_1 \). (They have the same magnitude and point in opposite directions.)

(e) \[ \vec{w}_3 + \vec{w}_4 = -\vec{v} \], by an analog of either of the two explanations in (c).
1.4, A7 (ab).

(c) \[
\begin{bmatrix}
2 & 1 \\
3 & 2 \\
1 & -2
\end{bmatrix}
\begin{bmatrix}
6 \\
2 \\
4
\end{bmatrix}
= 2
\begin{bmatrix}
3 \\
1 \\
2
\end{bmatrix},
\]
so \( \vec{a} \) and \( \vec{u} \) are parallel.

(b) Suppose \[
\begin{bmatrix}
-1 & 4 \\
6 & -6 \\
12 & 6
\end{bmatrix}
= c
\begin{bmatrix}
3 \\
1 \\
2
\end{bmatrix}
\]
for some scalar \( c \).

Comparing the first coefficient, \( c = -6 \). But comparing the third coefficient, \( c = 6 \). These can't both be true, so the vectors are not parallel.

Note. We also know that \( \vec{u} \) and \( \vec{v} \) are parallel if and only if \[
\frac{\vec{u} \cdot \vec{v}}{||\vec{u}|| ||\vec{v}||} = 1 \text{ or } -1.
\] You can also solve the problem this way.