Homework 7 - Math 544, Frank Thorne (thorne@math.sc.edu)

Due Monday, November 2, 2015 SUBJECT TO ADDITIONS.

(1) Do A 1 (a-c), 12-14 from Knop.

(2) For each of the Markov chains below:

(i) Describe in a couple of sentences what the transition matrix is telling you. You don't have to describe the matrix completely; just point out some of its most interesting features.

(ii) Compute the square of the transition matrix and briefly describe it.

(iii) Compute a steady state vector for the Markov chain.

(iv) For *one* of the Markov chains (your choice) with transition matrix M, compute M^3, M^4, and M^5. Describe these as well. Do the values appear to be converging to the steady state vector?

(a) This matrix predicts the outcome of a Gamecocks football win, based on the outcome of their previous game. The states are: Gamecocks win, Gamecocks lose.

$$\begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{bmatrix}$$

(b) Suppose you take one math class a semester. This matrix predicts your grade, based on the grade you earned in the previous course. The states are: A, B, C, D/F.

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 & 0.0 \\ 0.2 & 0.5 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.4 & 0.3 \\ 0.0 & 0.1 & 0.1 & 0.6 \end{bmatrix}$$

(c) This matrix predicts how well you sleep in a given night, based on how well you slept the previous night. The states are: good night of sleep, so-so, insomnia.

$$\begin{bmatrix} 0.7 & 0.2 & 0.5 \\ 0.2 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.3 \end{bmatrix}$$

(d) This matrix predicts how likely you are to go to the gym and exercise, based on whether or not you exercised the previous day. The states are: exercised, did not exercise.

$$\begin{bmatrix} 0.2 & 0.5 \\ 0.8 & 0.5 \end{bmatrix}$$

(e) This matrix predicts the day of the week. The states are Sun, M, T, W, Th, F, Sat.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(Hint: it is easier to find a steady state vector by thinking about it, rather than by a brute force computation.)

(f) Create your own! Describe a scenario which can be modeled by a Markov chain, write down a suitable transition matrix, and answer the same questions.

(g) (**Extra Credit 1**) The following Markov chain predicts whether or not today is Tuesday. The states are Tuesday and Not Tuesday.

$$\begin{bmatrix} 0 & \frac{1}{6} \\ 1 & \frac{5}{6} \end{bmatrix}$$

Writing M for the matrix, compute M^2. Explain why M is 'correct' but M^2 is 'wrong'. How can this be? Discuss.

(h) (**Extra Credit 2**) Here is a website that uses Markov chains to generate nonsense text:

`http://www.bitsofpancake.com/programming/markov-chain-text-generator/`

Here is a more elaborate website that does essentially the same thing, on a larger scale:

`https://pdos.csail.mit.edu/archive/scigen/`

The authors managed to get their nonsense accepted at computer science conferences, and they gave nonsense talks at them. You can generate your own!

Read these websites, experiment with them, and briefly summarize the basic idea of how they work.