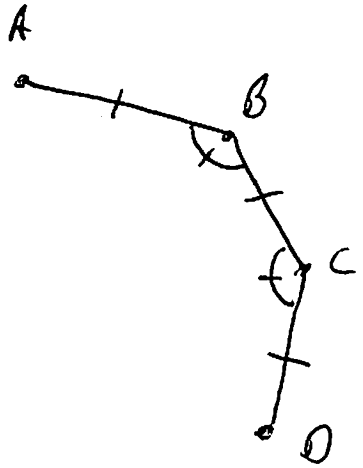
L6.2, 3.

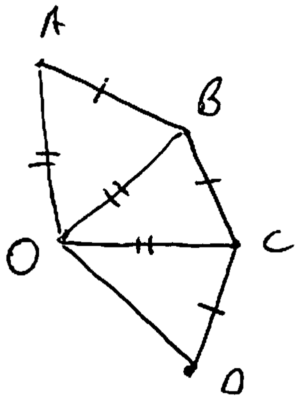
L6.2.



Suppose: A, B, C, D are 4 verts. (consec.) of a regular polygon.

Show. The circumcircle of ΔABC passes through D.

__Proof__: Let the circumcircle of ΔABC have center O.



$AO = BO = CO$ (radii)

$\overset{SSS}{\Longrightarrow} \Delta ABO \cong \Delta BCO$

$\xrightarrow[\text{parts}]{\text{corr}} \angle OBA = \angle OCB$.

But also, regularity $\Longrightarrow$ $\angle ABC = \angle BCD$. We have

$$\left.\begin{array}{l} \angle ABC = \angle OBA + \angle OBC \\ \angle BCD = \angle OCB + \angle OCD \end{array}\right\} \Longrightarrow \angle OBA + \angle OBC = \angle OCB + \angle OCD$$

$\xrightarrow{\angle OBA = \angle OCB} \angle OBC = \angle OCD$.

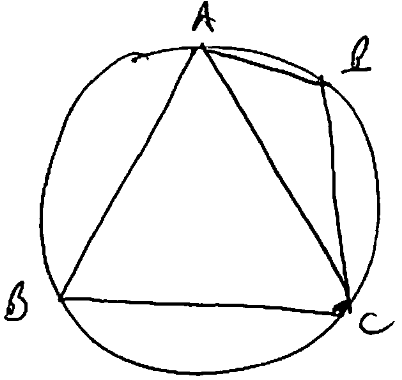
Consider:

$\underline{\Delta BCO}$		$\underline{\Delta CDO}$
BO	$=$	CO (radii)
$\angle OBC$	$=$	$\angle OCD$
BC	$=$	CD (regularity)

$\overset{SAS}{\Longrightarrow} \Delta BCO \cong \Delta CDO$

$\xrightarrow[\text{parts}]{\text{corr}} CO = DO \Longrightarrow D$ is on the circle.

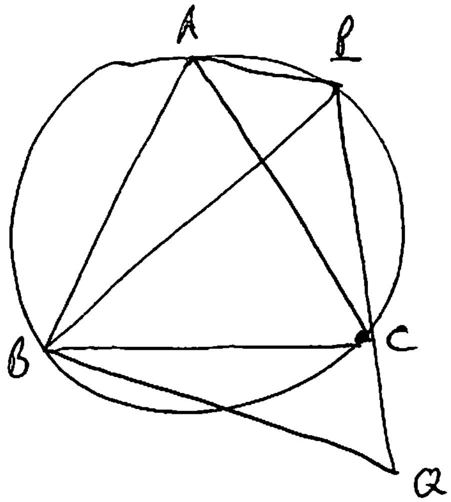
16.3



Suppose: $\triangle ABC$ is equilateral.

Show: $AP + PC = PB$.

<u>Proof:</u>



Draw PQ so that $PQ = PB$.

To show: $\triangle ABP \cong \triangle CBQ$.

(1) <u>Claim</u>: $\angle CQB = \angle APB$.

$\triangle ABC$ equil. $\Rightarrow \angle BAC = 60°$. But also, $\angle BAC \doteq \frac{1}{2}\widehat{BC} \doteq \angle BPC$,

so $\angle BPC = \angle BPQ = 60°$. Now, $BP = QP$ (hyp.) $\xRightarrow{p.a} \angle PBQ = \angle PQB$.

We have $\underbrace{\angle PBQ + \angle PQB}_{=2\angle PQB} + \underbrace{\angle BPQ}_{=60°} = 180° \Rightarrow \angle PQB = \angle CQB = 60°$.

We also have $\angle APB \doteq \frac{1}{2}\widehat{AB} \doteq \angle ACB = 60°$ since $\triangle ABC$ equil, so $\angle APB = 60°$.

(2) <u>Claim</u>: $\angle PAB = \angle QCB$.

Opp. angles in cyclic $APCB$ are supp. $\Rightarrow \angle PAB + \angle PCB = 180$.

But also, $\angle QCB + \angle PCB = 180°$ (straight angle)

$\Rightarrow \angle PAB = \angle QCB$

We now have: $\underline{\Delta ABP}$ $\underline{\Delta CBQ}$

$$\left.\begin{array}{rl} (1) \Rightarrow & \angle APB = \angle CQB \\ (2) \Rightarrow & \angle PAB = \angle QCB \\ \Delta ABC \text{ equil} \Rightarrow & AB = BC \end{array}\right\} \xrightarrow{AAS} \Delta ABP \cong \Delta CBQ$$

$$\underset{\text{parts}}{\xrightarrow{\text{corr}}} AP = CQ \xrightarrow{+PC} AP + PC = PC + CQ = PQ \underset{\substack{\uparrow \\ \text{hyp.}}}{=} PB$$

Thus, we conclude that $AP + PC = PB$.