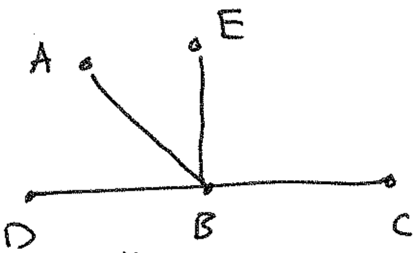
HW 5.

1. Euclid's argument (at least together with the picture) presumes A is on the same side of BE as C.

It fixes the argument to require this to be true. (Since we can switch C and D). But Euclid didn't specify this.

Otherwise, suppose we have this picture



Then $\angle CBE$ is not

$$\angle CBA + \angle ABE.$$

Starting with "Since the angle CBE..." we instead have:

$$\angle EBD = \angle EBA + \angle ABD$$

Add $\angle EBC$ to each.

Then $\angle EBD + \angle EBC = \angle EBA + \angle ABD + \angle EBC$

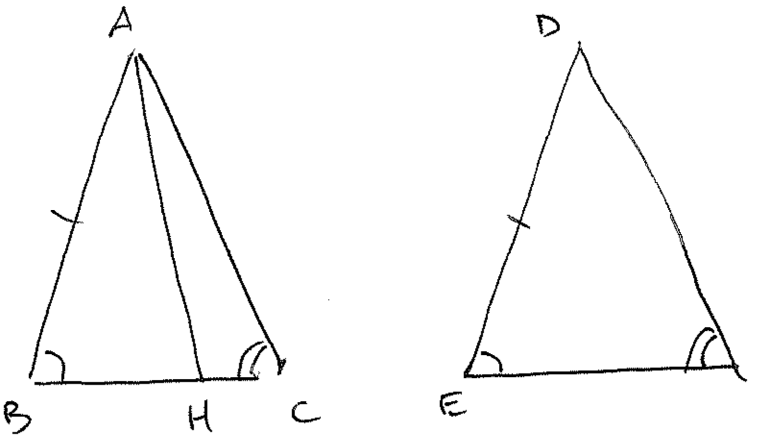
But also, since $\angle ABC = \angle ABE + \angle EBC$, add $\angle \cancel{EBE}\ ABD$ to each to get

$$\angle ABC + \angle ABD = \angle ABE + \angle EBC + \angle ABD.$$

Therefore $\angle ABC + \angle ABD = \angle EBD + \angle EBC$.

The latter is two right angles, so the sum of $\angle ABC$ and $\angle ABD$ makes two right angles. Q.E.D.

2. Suppose we have $\triangle ABC$ and $\triangle DEF$ with $\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$, and $AB = DE$.



If $BC \neq EF$, then without loss of generality assume $BC > EF$, and draw H on ~~EF~~ BC so that $EF = BH$.

By SAS $\triangle ABH \cong \triangle DEF$,

~~so that $\angle BAH = \angle EDF$. But~~

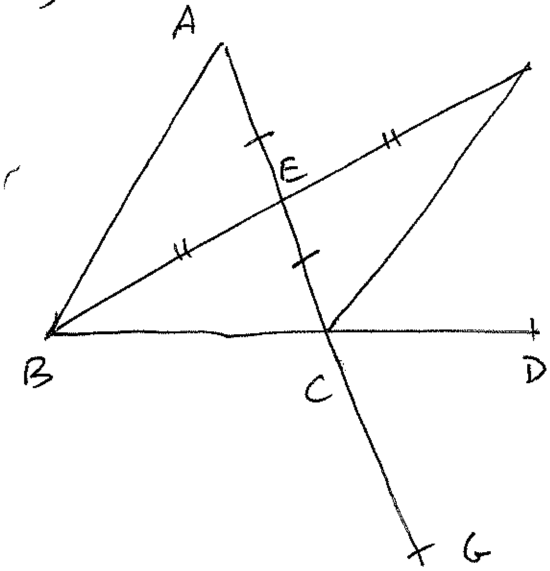
so that $\angle BHA = \angle EFD = \angle BCA$.

But this says the exterior angle $\angle BHA$ equals the interior and opposite angle $\angle C$ of $\triangle AHC$, proved impossible by Prop. 16.

Therefore $BC = EF$ and so by SAS

$\triangle ABC \cong \triangle DEF$.

3.



Given this picture from Prop. 16 with $\triangle ABE \cong \triangle CFE$.

In particular $\angle BAE = \angle ECF$.

By Prop. 27 $AB \parallel CF$.

But then BC is a transversal of the parallel lines AB and CF, and so Prop. 29 implies that $\angle ABC = \angle FCD$.

So, $\angle ABC + \angle BCA + \angle A$

$$= \angle FCD + \angle BCA + \angle ECF$$

$$= \angle FCD + \angle FCB$$

which is equal to two right angles by Prop. 13.

QED.

This is similar to the proof of Prop. 32. In Prop. 32, CF (or CE in Prop. 32) is constructed rather than proved to be parallel, and $\angle BAE = \angle ECF$ (in the picture above) is derived as a consequence of this.