Quiz 9 - Math 374, Frank Thorne (thorne@math.sc.edu)

Monday, April 6, 2015

1. How many integers are there from 1000 to 9999?

   Solution. The answer is
   $$9999 - 1000 + 1 = 9000.$$
   Another way to think about it: They are the integers from 1 to 9000 with 999 added to each, so clearly there are 9000 of them.

2. How many odd integers are there from 1000 to 9999?

   Solution. These are the numbers $2 \cdot 500 + 1$, $2 \cdot 501 + 1$, and so on through $2 \cdot 4999 + 1$. So the answer is
   $$4999 - 500 + 1 = 4500,$$
   or half of them.

   Alternate Solution. There are 9 possibilities for the first digit (1 through 9), 10 possibilities for the second digit (0 is now also allowed), 10 possibilities for the third digit, and 5 possibilities for the fourth digit. (1, 3, 5, 7, and 9) The total number of possibilities is
   $$9 \cdot 10 \cdot 10 \cdot 5 = 4500.$$

3. How integers from 1000 to 9999 have distinct digits?

   Solution. There are 9 possibilities for the first digit (1 through 9), 9 possibilities for the second digit (everything other than whatever you chose for the first digit), 8 possibilities for the third digit (everything other than your first two choices), and similarly 7 possibilities for the fourth digit.

   The total number of possibilities is
   $$9 \cdot 9 \cdot 8 \cdot 7 = 4536.$$

   In principle, other solutions are possible (e.g. count the number of integers with repeated digits), but these are *very* messy.