Quiz 5 - Math 374, Frank Thorne (thorne@math.sc.edu)

Monday, February 16, 2015

(1) (14 points) Let $D = E = \{-2, -1, 0, 1, 2\}$. Write negations for each of the following statements and determine which is true, the given statement or its negation.

(a) $\forall x$ in D, $\exists y$ in E such that $x + y = 1$.

Answer: $\exists x$ in D such that $\forall y$ in E, $x + y \neq 1$.

This follows the pattern of switching all the quantifiers and negating the statement in the middle. The negation is true: Take $x = -2$. Then, for each $y \in E$ we have that $x + y \leq 0$ and in particular $x + y \neq 1$.

(b) $\exists x$ in D such that $\forall y$ in E, $x + y = -y$.

Answer: $\forall x$ in D, $\exists y$ in E such that $x + y \neq -y$. The negation is true. The equation is equivalent to the equation $x = -2y$ and it is clear that no x can equal $-2y$ for multiple values of y simultaneously. Alternatively, one can list all five values of x in D, and for each of them come up with a y for which $x + y \neq -y$. (This is the one justification that will always work for statements of this form.)

Some people wrote $\sim (\exists x$ in D such that $\forall y$ in E, $x + y = -y)$. Strictly speaking, this *is* a negation but not a good answer. As done in class and the book, a good answer is in the form where the negation is 'passed to the middle', so to speak.

(2) (6 points) Indicate whether the argument is valid by universal modus ponens or universal modus tollens, or exhibits the converse or the inverse error.

All honest people pay their taxes.

Darth is not honest.

$\therefore$ Darth does not pay his taxes.

This is *invalid*, and exhibits the *universal inverse error.*

This is not the converse error (although not too many points were taken off for mistaking the inverse error for the converse error); that would be if the second premise read 'Darth pays his taxes' and the conclusion was 'Darth is honest'. If the first premise is rewritten in terms of the contrapositive ('All people who do not pay their taxes are dishonest') then this would be an instance of the converse error.

The question did not ask you to write out the logical form of the argument. Most of those of you who did so did it incorrectly. Points were not taken off for this, since it was not part of the question. But, for reference, the correct form is

$$\forall x \; H(x) \to T(x)$$
$$\sim H(D)$$
$$\therefore \sim T(D).$$