Quiz 4 - Math 374, Frank Thorne (thorne@math.sc.edu)

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(1) Let D be the set of all USC students, let $M(s)$ be "s is a math major", let $C(s)$ be "s is a computer science student", and let $E(s)$ be "s is an engineering student". Express each of the following statements using quantifiers, variables, and the predicates $M(s)$, $C(s)$, and $E(s)$.

(a) There is an engineering student who is a math major.

Solution: $\exists x \in D,\ E(x) \wedge M(x)$. In particular, the sentence has the same literal meaning (with a different emphasis) as 'There is a math major who is an engineering student.'

(b) Some computer science students are also math majors.

Solution: $\exists x \in D,\ C(x) \wedge M(x)$.

(c) Some computer science students are engineering students and some are not.

Solution: $(\exists x \in D,\ C(x) \wedge M(x)) \wedge (\exists x \in D,\ C(x) \wedge \sim M(x))$.

In particular, note that the statement describes the existence of two different kinds of students. Therefore it needs multiple $\exists$ quantifiers to express.

(2) Write negations for each of the following statements.

(a) $\forall$ computer programs P, if P compiles without error messages, then P is correct.

Solution: There exists a computer program P, such that P compiles without error messages and P is not correct.

A useful intermediate step is: There exists a computer program P, such that NOT (if P compiles without error messages, then P is correct). Then one remembers what the negation of an implication is.

(b) $\forall x \in \mathbb{R}$, if $x(x+1) > 0$ then $x > 0$ or $x < -1$.

Solution: $\exists x \in \mathbb{R},\ x(x+1) > 0 \wedge (x \leq 0) \wedge (x \geq -1)$. It is also correct to put English 'and' in place of the $\wedge$ symbols. Yet another correct variation replaces the two inequalities with $-1 \leq x \leq 0$. (Indeed, this is probably best.)

To come up with this, it might help again to write out the intermediate step:

$\exists x \in \mathbb{R}, \sim($ if $x(x+1) > 0$ then $x > 0$ or $x < -1)$.