

Quiz 3 - Math 374, Frank Thorne (thorne@math.sc.edu)

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(1) Write in if-then form:

(a) Catching the 8:05 bus is a sufficient condition for my being on time to work.

Solution: If I catch the 8:05 bus, then I will be on time to work.

Alternate correct solutions also exist, but this is the most natural and simple solution.

(b) Doing homework regularly is a necessary condition for Jim to pass the course.

Solution: If Jim passes the course, then he did his homework regularly.

(One can also tweak with the tenses, for example: *If Jim will pass the course, then he will have done his homework regularly.* The important thing is to get the if-then in the right direction.)

Alternate solution: If Jim doesn't do his homework, then Jim won't pass the course. (Any conditional is logically equivalent to its contrapositive, so this is equally correct.)

(2) A set of premises and a conclusion are given. Use valid argument forms to deduce the conclusion from the premises, giving a reason for each step.

(a) $\sim p \rightarrow r \wedge \sim s$

(b) $t \rightarrow s$

(c) $u \rightarrow \sim p$

(d) $\sim w$

(e) $u \vee w$

(f) $\therefore \sim t$

(See next page for solution)

Solution: A formal proof can be given as follows.

$$\begin{array}{l} u \rightarrow \sim p \\ \sim p \rightarrow r \wedge \sim s \\ \therefore u \rightarrow r \wedge \sim s \text{ (transitivity)} \end{array}$$

$$\begin{array}{l} u \vee w \\ \sim w \\ \therefore u \text{ (elimination)} \end{array}$$

$$\begin{array}{l} u \\ u \rightarrow r \wedge \sim s \\ \therefore r \wedge \sim s \text{ (modus ponens)} \end{array}$$

$$\begin{array}{l} r \wedge \sim s \\ \therefore \sim s \text{ (specialization)} \end{array}$$

$$\begin{array}{l} \sim s \\ t \rightarrow s \\ \therefore \sim t \text{ (modus tollens)} \end{array}$$

Other correct solutions are possible; for example it is possible to use modus ponens twice instead of transitivity and modus ponens once.

Note: On exams, Table 2.3.1 of valid argument forms will be given to you and you will be expected to reproduce a proof in this style. In particular, you will be expected to understand and use the argument forms given there, but you are not responsible for remembering which forms are and are not on the list, nor for remembering their formal names.