Quiz 12 - Math 374, Frank Thorne (thorne@math.sc.edu)

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1. Does there exist a graph with four vertices of degree 1, 2, 3, and 4? Either draw one or explain why no such graph exists.

    Solution. Such a graph does exist. (Indeed, there are several non-isomorphic such graphs.) For example, label the vertices v_1, v_2, v_3, and v_4; draw one edge between v_1 and v_2, one edge between v_2 and v_3, two edges between v_3 and v_4, and a loop from v_4 to itself.

    Note that loops *are* allowed, unless said otherwise. (If the question asks for a *simple* graph, then loops and parallel edges are not allowed.) So solutions which tried to argue (for example) that no vertex can have degree 4, because there are only three other vertices to connect to, are incorrect.

2. Determine which of the two graphs below have Euler circuits. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one.

    Recall that an Euler circuit is a walk which starts and ends at the same vertex, and contains every edge exactly once.

    Solution. The pictures were the graphs in Problems 12 and 13 from 10.2 of Epp.

    The first graph does have an Euler circuit. For example, start at v_1, walk along the edges e_1 through e_8 in numerical order, and finish at v_1. There are many other correct answers as well.

    The second graph does not, because some of the vertices (v_9, for example) have odd degree.

    Note that, for the second part, it is not a correct justification to say something like "The graph does not have an Euler circuit because whatever path you try to take, you'd have to repeat an edge." This is essentially repeating the definition of an Euler circuit, and so this is circular reasoning.