Quiz 11 - Math 374, Frank Thorne (thorne@math.sc.edu)

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1. (a) How many 9-bit strings contain exactly seven 1's?

   Solution. Out of nine places, you must pick 7 of them to contain 1's, and the other two will contain 0's. The answer is

$$\binom{9}{7} = \frac{9 \cdot 8}{2 \cdot 1} = 36.$$

   (b) How many 9-bit strings contain at least seven 1's?

   Solution. Such a bit string can contain 7, 8, or 9 ones. The answer is

$$\binom{9}{7} + \binom{9}{8} + \binom{9}{9} = 36 + 9 + 1 = 46.$$

   (c) How many 9-bit strings contain at least one 1?

   Solution. It is possible to solve this as above, namely the answer is equal to $\sum_{i=1}^{9} \binom{9}{i}$. There is a lot of arithmetic to do and you eventually find that the answer is 511.

   An easier solution is as follows. Without restriction on the zeroes and ones, a total of $2^9 = 512$ bit strings are possible. Of these, you exclude only the one with all zeroes, so the total number of allowed strings is $512 - 1 = 511$.

2. An instructor gives an exam with 10 questions. Students are allowed to choose any six to answer.

   (a) How many different choices of six questions are there?

   Solution.

$$\binom{10}{6} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210.$$

   (b) Suppose that five questions require proof and five do not. How many groups of six questions contain at most three that require proof?

   Solution. Suppose, first, that you answer exactly three that require proof. Then this leaves three to answer that do not require proof. The total number of possibilities is

$$\binom{5}{3} \cdot \binom{5}{3} = 10 \cdot 10 = 100.$$

   Similarly, you might answer exactly two or one that require proof. (You can't answer none, because then you would need to answer six other questions, but there are only five to choose from.) The total number of these possibilities is

$$\binom{5}{2} \cdot \binom{5}{4} + \binom{5}{1} \cdot \binom{5}{5} = 10 \cdot 5 + 5 \cdot 1 = 55.$$

So the total is 155.

Alternate solution. The total number of groups of questions, where four or five questions require proof, is (by similar reasoning)

$$\binom{5}{4} \cdot \binom{5}{2} + \binom{5}{5} \cdot \binom{5}{1} = 5 \cdot 10 + 1 \cdot 5 = 55.$$

These are the disallowed solutions. Subtracting these from the first part of the question, the total number of possibilities is $210 - 55 = 155$.