

Quiz 10 - Math 374, Frank Thorne (thorne@math.sc.edu)

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1. How many integers from 1 through 1000 are neither multiples of 4 nor multiples of 7?

Solution. There are $\lfloor \frac{1000}{4} \rfloor = 250$ multiples of 4. There are $\lfloor \frac{1000}{7} \rfloor = 142$ multiples of 7. Finally, there are $\lfloor \frac{1000}{28} \rfloor = 35$ multiples of 28. (A number is a multiple of both 4 and 7 if and only if it is a multiple of 28. Note also that a shortcut to computing this is to compute $\lfloor \frac{142}{4} \rfloor$.) So, the number of integers between 1 through 1000 which are multiples of 4 or 7 or both is

$$250 + 142 - 35 = 357$$

by the Inclusion-Exclusion Principle. Therefore the answer is $1000 - 357 = 643$.

2. In a group of 2,000 people, must at least 5 have the same birthday? Why?

Solution. Yes. There are 366 different days of the year. As soon as you have $4 \cdot 366 + 1 = 1465$ people, there must be some day on which $\lceil \frac{1465}{366} \rceil = 5$ people were born.

This is still true if you have more than 1465 people. Indeed, with 2,000, at least 6 must have the same birthday, by similar reasoning.