Quiz 6 - Math 374, Frank Thorne (thorne@math.sc.edu)

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(1) A function $T(n)$ is defined on integer powers of 2 by the following recurrence:

- $T(1) = 3$, and
- $T(n) = T(n/2) + n$ when $n \geq 2$ and $n = 2^m$ for some integer m.

Obtain a closed-form formula for this recurrence. Prove your claim.

Solution. By using the recurrence, we find that: $T(1) = 3$, $T(2) = 5$, $T(4) = 9$, $T(8) = 17$, $T(16) = 33$, and so on. By pattern matching we guess that $T(n) = 2n + 1$.

We prove this by induction. The base case is true, because $T(1) = 3 = 1 \cdot 1 + 1$. So assume that $T(k) = 2k + 1$ for some integer n which is at least 2, and a power of 2. We must prove that $T(2k) = 2(2k) + 1 = 4k + 1$.

We have, by the recursive definition,

$$T(2k) = T(k) + 2k = 2k + 1 + 2k = 4k + 1,$$

as desired, so the claim follows.