Inference Rules (corrected) - Math 374, Frank Thorne (thorne@math.sc.edu)

You may print out a copy of this sheet and bring and use it on quizzes and exams.

Equivalence Rules:

1. **Commutative:** $P \wedge Q$ is equivalent to $Q \wedge P$; $P \vee Q$ is equivalent to $Q \vee P$.
2. **Associative**: $(P \wedge Q) \wedge R$ is equivalent to $P \wedge (Q \wedge R)$; $(P \vee Q) \vee R$ is equivalent to $P \vee (Q \vee R)$.
3. **De Morgan**: $(P \vee Q)'$ is equivalent to $P' \wedge Q'$; $(P \wedge Q)'$ is equivalent to $P' \vee Q'$.
4. **Implication**: $P \to Q$ is equivalent to $P' \vee Q$.
5. **Double negation**: P is equivalent to $(P')'$.
6. **Equivalence**: $P \leftrightarrow Q$ is equivalent to $(P \to Q) \wedge (Q \to P)$.

Inference Rules (Gersting, p. 25):

1. **Modus ponens**: From P and $P \to Q$, you may conclude Q.
2. **Modus tollens**: From $P \to Q$ and Q', you may conclude P'.
3. **Conjunction**: From P and Q, you may conclude $P \wedge Q$.
4. **Simplification**: From $P \wedge Q$, you may conclude P and Q.
5. **Addition**: From P, you may conclude $P \vee Q$.

Three More Inference Rules:

1. **Elimination**: From $P \vee Q$ and Q', you may conclude P. From $P \vee Q$ and P', you may conclude Q.
2. **Transitivity**: From $P \to Q$ and $Q \to R$, you may conclude $P \to R$.
3. **Cases:** From $P \vee Q$, $P \to R$, and $Q \to R$, you may conclude R.

Inference Rules with Quantifiers (Gersting, p. 50):

1. **Universal instantiation**: From $(\forall x)P(x)$, you can derive $P(t)$, where t is a variable or constant symbol.

   Restriction: If t is a variable, it must not fall within the scope of a quantifier for t.

2. **Existential instantiation**: From $(\exists x)P(x)$, you can derive $P(a)$, where a is a constant symbol not previously used in proof sequence.

   Restriction. Must be the first rule used that introduces a.

3. **Universal generalization**: From $P(x)$, you can derive $(\forall x)P(x)$.

   Restriction: (1) $P(x)$ has not been deduced from any hypotheses in which x is a free variable. (2) $P(x)$ has not been deduced by existential instantiation from any wff in which x is a free variable.

4. **Existential generalization**: From $P(x)$, or from $P(a)$ where a is a constant symbol, you can derive $(\exists x)P(x)$.

   Restriction: To go from $P(a)$ to $(\exists x)P(x)$, x must not appear in $P(a)$.