Final Exam Review - Math 142, Frank Thorne (thornef@mailbox.sc.edu)

The exam is cumulative. *Warning:* Just because a flavor of problem doesn't appear here doesn't mean it won't appear on the final exam! If your memory of a subject is hazy, you are strongly advised to go to the relevant section of the book and do more of the exercises.

Part 1. Techniques of Integration.

(1) What is the rule for integration by substitution, and which differentiation rule does it follow from? Explain.

(2) What is the rule for integration by parts, and which differentiation rule does it follow from? Explain.

(3) What is an improper integral? Explain the different ways in which an integral can be improper and how you evaluate improper integrals of this form.

(4) Evaluate
$$\int \sqrt{x}\sin(1+x^{3/2})dx.$$

(5) Evaluate
$$\int_0^4 \frac{x}{\sqrt{1+2x}}dx.$$

(6) Evaluate
$$\int \sin(\sqrt{x})dx.$$

(7) Evaluate
$$\int_1^2 \frac{(\ln x)^2}{x^3}dx.$$

(8) Evaluate
$$\int \sin^2(x)dx.$$

(9) Evaluate
$$\int_{\sqrt{2}}^2 \frac{1}{t^3\sqrt{t^2-1}}dt.$$

(10) Evaluate
$$\int \frac{\sqrt{1+x^2}x}{d}x.$$

(11) Evaluate
$$\int \frac{x+4}{x+5}dx.$$

(12) Evaluate
$$\int \frac{x-6}{x^2-6x+5}dx.$$

(13) Evaluate
$$\int_{-\infty}^{\infty} \frac{4}{1+9x^2} dx.$$

(14) Evaluate
$$\int_0^{33} (x-1)^{-1/5} dx.$$

(15) Evaluate
$$\int_0^{\infty} \frac{x}{(x^2+2)^2} dx.$$

Part 2. Applications of integration.

1. Explain the formulas for the **area under a curve**, the **volume of a solid of revolution**, and the **arc length of a curve**. What are the correct formulas, and why are they true? Explain thoroughly.

2. Determine the area bounded by the curves $y = \cos x$ and $y = 2 - \cos x$ for $0 \leq x \leq 2\pi$.

3. Determine the area bounded by the curves $4x + y^2 = 12$ and $x = y$.

4. Find the volume of the solid obtained by rotating the region bounded by the curves
$$y = \ln(x), y = 1, y = 2, x = 0$$
around the y-axis. Sketch the region, the solid, and a typical disk or washer.

5. Find the volume of the solid obtained by rotating the region bounded by the curves
$$y = \frac{1}{4}x^2, y = 5 - x^2$$
around the x-axis. Sketch the region, the solid, and a typical disk or washer.

6. Find the volume of a sphere of radius r.

7. Find the volume of a cap of a sphere of radius r and height h. (See 6.2, 51 for a picture.)

8. Find the length of the curve
$$y = e^x$$
for $0 \leq x \leq 1$.

Part 3. Parametric and polar coordinates.

1. What are parametric and polar coordinates? Give examples of their use.

2. Give examples of equations in (1) parametric, and (2) polar coordinates that (a) can easily be, (b) cannot easily be converted into Cartesian coordinates.

3. Sketch the parametric curve given by the equations

$$x = 2\cos(t), y = t - t\cos(t)$$

for $0 \le t \le 2\pi$.

4. Review exercises 24-28 in Section 10.1 in your book, and also 49, 50, and 56 of Section 10.3. (The ones with the pictures you have to match or draw.)

5. Recall the statement of the 'bug problem'. Formulate several versions and in each case give parametric equations describing the motion of the bug. In addition, draw the curve described by the bug's motion and compute the tangent line at representative points.

6. Find the points on the curve where the tangent is horizontal or vertical. Graph the curve and check your work.

$$x = 10 - t^2, y = t^3 - 12t$$

7. Sketch the curve $r = 2 + \sin\theta$, and find and draw the tangent line (in Cartesian coordinates) at $\theta = \frac{\pi}{4}$. Moreover, estimate the area contained inside this curve (based on your graph) and then compute it exactly.

8. Find the area of the region that lies betwen the curves $r = 3 + 2\cos\theta$ and $r = 3 + 2\sin\theta$. Also, sketch both curves and explain why your answer matches your graph.

Part 4. Infinite sequences and series. (Refer to the list of sequences and series on the website and distributed with the third midterm. You will get the same sheet on the final exam.)

1. What are infinite sequences and series? What does it mean for them to converge? Explain thoroughly.

2. Determine a general formula for the sequence

$$-2, 3, -2, 3, \cdots$$

Come up with a formula that doesn't divide into separate cases. For example, $a_n = -2$ if n is odd and $a_n = 3$ if n is even would otherwise be an acceptable solution, but you are asked for a solution not using such language.

3. For what values of r is the formula

$$a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1-r}$$

true? Why is is true for these r, and why is it not true for other r?

4. Evaluate

$$\sum_{n=1}^{\infty} \frac{2}{n^2 + 4n + 3}.$$

5. Using the integral test, determine whether the series below converge or diverge. In each case draw a picture. When the series converges, compute lower and upper bounds which are accurate to within 0.1, draw two pictures which illustrate your lower and upper bounds respectively, and use your picture to explain your bounds.

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^{1/2}}}.$$

$$\sum_{n=3}^{\infty} \frac{1}{n^2+9}$$

6. Determine whether the series converge or diverge:

$$\sum_{n=1}^{\infty} \frac{1}{2n+3}$$

$$\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{n+5}{(n^7+n^2)^{1/3}}$$

7. Determine whether the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$$

converges or diverges. If it converges, approximate it within 0.1.

8. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n!}{100^n}$$

converges or diverges.

9. Compute the Taylor series for each of the following functions:

$$\sin(2x), e^x + e^{2x}, \tan^{-1}(3x), \frac{1}{1-2x}$$

In each case use the ratio test to determine the radius and interval of convergence.

10. Compute the Taylor series of $(1+x)^{1/3}$ through the x^3 term. Use your computation to come up with a good approximation to the cube root of 1.09, and come up with an estimate (which you do not need to justify rigorously) for the accuracy of your approximation.