(a) What is a Taylor series? Why is the formula for it true?

(b) Find the Maclaurin (Taylor) series for the following functions. Determine their radii of convergence.

- \( f(x) = x^2 \)
- \( f(x) = e^x \)
- \( f(x) = e^{2x} \)
- \( f(x) = \cos(x) \)
- \( f(x) = \sin(x) \)
- \( f(x) = \cos(4x) \)
- \( f(x) = \sin(x^2) \)
- \( f(x) = x^3 \sin(x) \)
- \( f(x) = x + e^x \).

(c) Explain why the Taylor series for \( e^x \) gives you a formula for \( e \).

(d) Compute \( e \), as a fraction or decimal, to fairly good accuracy. Your estimate should plausibly be within \( \frac{1}{10} \), but you don’t need to show this.

(e) Compute \( 1/e \), as a fraction or decimal, to fairly good accuracy. Your estimate should plausibly be within \( \frac{1}{100} \), but you don’t need to show this.

(f) Compute \( \sin(1/10) \), as a fraction or decimal, to fairly good accuracy. Your estimate should plausibly be within \( \frac{1}{100} \).

(g) Compute \( \sqrt{1.1} \), as a fraction or decimal, to fairly good accuracy. Your estimate should plausibly be within \( \frac{1}{1000} \), but you don’t need to show this.

(h) Stewart, 11.10, 29, 30.

Additional problems:

(a) Stewart, 11.10, 15, 16, 31-36.

Bonus: Use Taylor series to explain why \( e^{ix} = \cos(x) + i\sin(x) \).