1. If $f(x) = \tan(x) = \dfrac{\sin(x)}{\cos(x)}$

$$\frac{df}{dx} = \frac{\cos x \frac{d}{dx}(\sin x) - \sin(x) \frac{d}{dx} \cos(x)}{\cos^2(x)}$$

$$= \frac{\cos x \cdot \cos x - \sin(x) \cdot (-\sin x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x).$$

If $f(x) = \csc(x) = \dfrac{1}{\sin(x)}$,

$$\frac{df}{dx} = \frac{\sin(x) \frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}(\sin x)}{\sin^2(x)}$$

$$= \frac{-\cos x}{\sin^2 x} = -\cot(x)\csc(x).$$

2. If $y = \ln(7 + 2x^5)$

then $\dfrac{dy}{dx} = \dfrac{1}{7+2x^5} \cdot \dfrac{d}{dx}(7 + 2x^5)$

$$= \frac{1}{7+2x^5}(10x^4) = \frac{10x^4}{7+2x^5}.$$

3. Suppose that $\frac{dy}{dt} = k \cdot y$

This is a differential equation that says the rate of change of something is proportional to the amount there. This models population growth, interest on money, heating and cooling, and other phenomena. We have $k > 0$ for growth and $k < 0$ for decay.

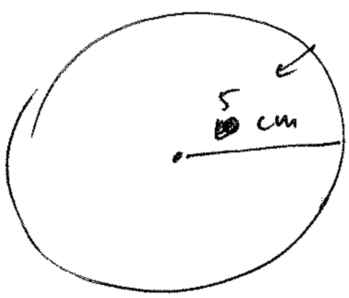
The solution is $y = C \cdot e^{kt}$.

To find c, if we plug in $t = 0$, we get $y(0) = c$.

So this becomes $y(t) = y(0)\, e^{kt}$.

$y(0)$ represents the initial amount.

4.



We have $S = 4\pi r^2$.

So $\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$.

When the diameter is 10 cm the radius is 5 cm, and

$$\frac{dS}{dt} = 40\pi \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{dS}{dt} \cdot \frac{1}{40\pi}$$

$$= (-1) \cdot \frac{1}{40\pi}$$

$$= \frac{-1}{40\pi}\ \mathrm{cm/s}.$$

So the diameter decreases at twice this rate, $\frac{1}{20\pi}$ cm/s.

5. Find the maximum and minimum of $\frac{x}{x^2+1}$ on $[0,2]$.

$$\frac{df}{dx} = \frac{(x^2+1)\cdot 1 - x\cdot(2x)}{x^2+1}$$

$$= \frac{x^2+1-2x^2}{x^2+1}$$

$$= \frac{-x^2+1}{x^2+1}.$$

This is always defined. It is 0 if $x^2=1$. On this interval $x=1$. (only)

So the critical points are $x=0,1,2$.

If $x=0$, $f(x) = \frac{0}{0^2+1} = 0$

If $x=1$, $f(x) = \frac{1}{1^2+1} = \frac{1}{2}$

If $x=2$, $f(x) = \frac{2}{2^2+1} = \frac{2}{5}$

So the minimum is $(0,0)$ and the maximum is $(1,\frac{1}{2})$.

6. $\frac{dy}{dx} = 4x - 4x^3.$

$\frac{d^2y}{dx^2} = 4 - 12x^2.$

These are always defined.

$\frac{dy}{dx} = 0?$

$4x - 4x^3 = 4x(1-x^2)$
$= 4x(1-x)(1+x)$

so $= 0$ if $\boxed{x = 0, 1, -1.}$
(critical points)

When is $\frac{d^2y}{dx^2} = 0?$ when $12x^2 = 4$

so $x^2 = \frac{1}{3}$

$x = \frac{\pm 1}{\sqrt{3}}.$

Plug in these points:

x	y
0	2
1	$2+2-1=3$
-1	$2+2-1=3$
$1/\sqrt{3}$	$2+\frac{2}{3}-\frac{1}{9} = \frac{18+6-1}{9} = \frac{23}{9}$
$-1/\sqrt{3}$	$\frac{23}{9}.$

Sign of derivative: if $x < -1$, take $x = -2$:

$4(-2) - 4(-2)^3$
$= -8 + 32 > 0.$

If $x \in (-1, 0)$ take $x = \frac{-1}{2}$:

$4\left(\frac{-1}{2}\right) - 4\left(\frac{-1}{2}\right)^3$
$= -2 + \frac{4}{8} < 0.$

If $x > 1$, take $x = 2$: $4(2) - 4(2)^3$
$= 8 - 32 < 0.$

if $x \in (0,1)$ take $x = \frac{1}{2}$: $4\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$
$= 2 - \frac{4}{8} > 0.$

Sign of $\frac{d^2y}{dx^2}$: if $y < \frac{-1}{\sqrt{3}}$:

Take $y = -1$, $4 - 12(-1)^2 < 0$

$y = 1$ $\Rightarrow$ same thing

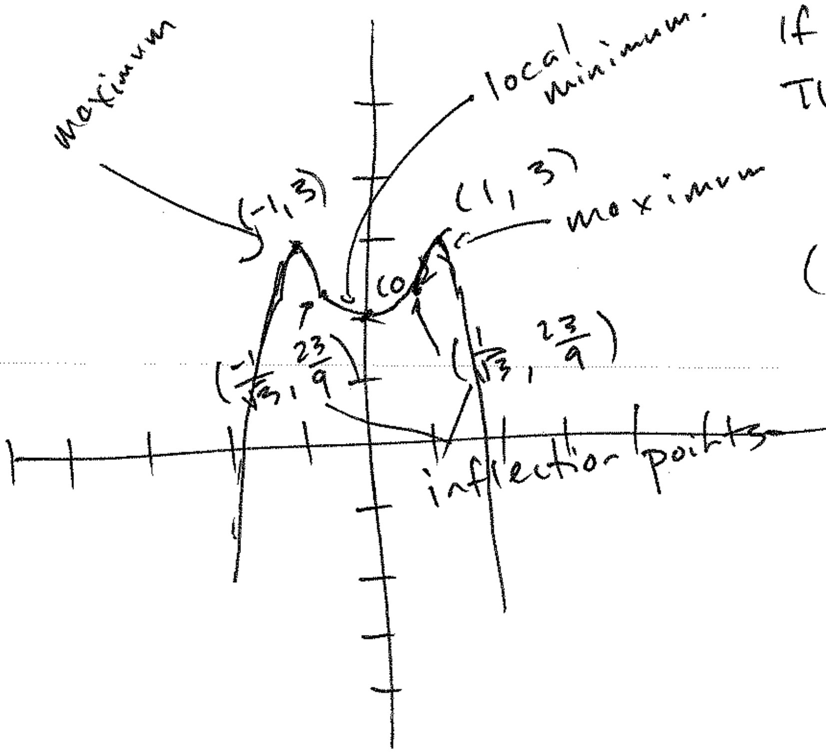
$y = 0$, get 4.

So: increasing in $(-\infty, -1)$, $(0, 1)$

decreasing in $(-1, 0)$, $(1, \infty)$.

concave up: $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

concave down: $\left(-\infty, \frac{-1}{\sqrt{3}}\right)$, $\left(\frac{1}{\sqrt{3}}, \infty\right)$.



If $x = 2$?
Then $y = 2 + 8 - 16 = -6$.
Same for -2.

(local maxima and minimum are the critical points)