Homework 6 - Math 141, Frank Thorne (thornef@mailbox.sc.edu)

Due Friday, October 12

Required problems:

(a) If $y = \ln x$, explain why $\frac{dy}{dx} = \frac{1}{x}$.

(b) Find $\frac{dy}{dx}$ if $y = \log_a x$.

(c) Explain why $e = \lim_{x \to 0}(1 + x)^{1/x}$.

(d) Explain why $e = \lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^n$.

(e) Stewart, Ch. 3.6, 1.

(f) Stewart, Ch. 3.6, 3-14, 37-42 (even).

(g) Stewart, Ch. 3.6, 49, 50.

(h) Stewart, Ch. 3.7, 5, 6, 7, 10, 12(a), 20, 21.

(i) Explain what kind of situations are natural modeled by an exponential decay law. Describe a general principle and give at least two examples.

(j) Explain the meaning and origin of the equation $y(t) = y(0)e^{kt}$. When is k positive, and when is it negative?

(k) What is a *differential equation*?

(l) Stewart, Ch. 3.8, 2, 8, 14, 18, 20.

Additional problems:

(a) Stewart, Ch. 3.6, 3-14, 37-42 (odd).

(b) Stewart, Ch. 3.7, 9, 14.

(c) Stewart, Ch. 3.8, 3, 9, 15.

Bonus (2 points). Say you take out a 30-year **mortgage** of $100,000 at 5%. This means that you pay interest of 5% on your debt, compounded continuously, and you make a monthly payment at the end of each month.

The monthly payment is calculated so that (1) it is the same every month, and (2) it will take you exactly 360 payments to pay off your entire mortgage (including principal and interest).

Compute the monthly payment. You may need to use a computer or calculator, but fully explain what you are doing.