

Final Examination (Version 5) - Math 141, Frank Thorne (thornef@mailbox.sc.edu)

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Please work without books, notes, calculators, or any assistance from others. If you have any questions, feel free to ask me. Please do your work on separate paper; you should staple this sheet to your work (put this on top) and turn in everything together.

All questions count equally.

- (1) Explain the meaning and origin of the equation $y(t) = y(0)e^{kt}$. When is it positive, and when is it negative?
- (2) What does the Fundamental Theorem of Calculus say? State both parts, and also include a picture, and/or explanation as appropriate.
- (3) (Do problem 7 on p. 410.)
- (4) Find $\frac{dy}{dx}$ if $y = \sqrt{x}(x - 1)$.
- (5) Find $\frac{dy}{dx}$ if $y = 2 \csc x + 5 \cos x$.
- (6) Evaluate $\int_{\pi}^{2\pi} \cos \theta d\theta$.
- (7) Evaluate $\int_0^2 y^2 \sqrt{1 + y^3} dy$.
- (8) Graph $f(x) = x^4 + 4x^3$. Explicitly describe each of the following: x and y -intercepts; where the graph is positive and negative; critical points; where increasing and decreasing; inflection points; where concave up and concave down; asymptotes if any.
- (9) A farmer wants to build a rectangular fence along a river. She has 120 feet of fencing, and she does not need any fence along the river. What is the most area she can enclose?
- (10) Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is $F = \frac{GmM}{r^2}$, where G is a constant and r is the distance between the bodies.
Find $\frac{dF}{dr}$ and explain its meaning. What does the minus sign indicate?
Suppose it is known that the earth attracts an object with a force that decreases at the rate of 2 N/km when r is 20,000 km. How fast does this force change when $r = 10,000$ km?
- (11) Sketch the region enclosed by the curves $y = x^2 - 2x$ and $y = x + 4$, and find its area. Draw a typical slice (i.e., rectangle) of the area.
- (12) Find the absolute maximum and minimum of $f(x) = \frac{x}{x^2+1}$ on $[0, 2]$.
- (13) (Do 2.7, 47.)

- (14) Find the derivative of $f(x) = \frac{1}{2}x - \frac{1}{3}$ using the definition of the derivative. State the domain of the function and the domain of the derivative.

Do not use any differentiation rules to get your answer (although this will be helpful to double check it).

- (15) (Do 2.8, 41.)

- (16) A bacteria population starts with 400 bacteria and grows at a rate of $450e^{1.1t}$ bacteria per hour. How many bacteria will there be after three hours?