

Final Examination (Version 4) - Math 141, Frank Thorne (thornef@mailbox.sc.edu)

Thursday, December 13, 2012

Please work without books, notes, calculators, or any assistance from others. If you have any questions, feel free to ask me. Please do your work on separate paper; you should staple this sheet to your work (put this on top) and turn in everything together.

All questions count equally.

- (1) What is a definite integral? Explain thoroughly and draw a picture.
- (2) Explain what kind of situations are naturally modeled by an exponential decay law. Describe a general principle and give at least two examples.
- (3) (Do Ch. 2.7, 11.)
- (4) Find $\frac{dy}{dx}$ if $y = \frac{(x+\lambda)^4}{x^4+\lambda^4}$. (Here λ is a constant.)
- (5) Find the first and second derivatives of the function $y = e^{e^x}$.
- (6) Evaluate $\int (1 + \tan^2 \alpha) d\alpha$.
- (7) Evaluate $\int_1^2 \frac{e^{1/x}}{x^2} dx$.
- (8) Graph $f(x) = \frac{\sin x}{1+\cos x}$. Explicitly describe each of the following: x and y -intercepts; where the graph is positive and negative; critical points; where increasing and decreasing; inflection points; where concave up and concave down; asymptotes if any.
- (9) A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus, the diameter of the semicircle is equal to the width of the rectangle.)
If the perimeter of the window is 30 ft, find the dimensions of the window allowing for the largest area.
- (10) Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2 ?
- (11) The region bounded by the curves $y = 2 - x/2$, $y = 0$, $x = 1$, and $x = 2$ is revolved around the x -axis. Sketch the region, the solid, and a typical slice, and compute the volume of the solid.
- (12) Evaluate $\int_0^4 \sqrt{4x - x^2}$ using geometry.
- (13) Let $K(t)$ be a measure of the knowledge you gain by studying for a test for t hours. Which do you think is larger, $K(8) - K(7)$ or $K(3) - K(2)$? Is the graph of K concave upward or concave downward? Why?
- (14) Graph the function $f(x) = x \tan x$ from $-\pi/2 < x < \pi/2$. Indicate on your graph any asymptotes, intervals of increase or decrease, local maxima and minima, intervals of concavity, and inflection points.

- (15) A mass on a spring vibrates horizontally on a smooth level surface. (See the picture on p. 196.) Its equation of motion is $x(t) = 8 \sin t$, where t is in seconds and x is in centimeters.
- Find the velocity and acceleration at time t . Also, find the position, velocity, and acceleration of the mass $t = 2\pi/3$. In what direction is it moving at that time?
- (16) By finding slopes of appropriate secant lines, determine (approximately) the equation of the tangent line to $y = x^2$ at $(1, 1)$.