Final Examination (Version 3) - Math 141, Frank Thorne (thornef@mailbox.sc.edu)

Thursday, December 13, 2012

Please work without books, notes, calculators, or any assistance from others. If you have any questions, feel free to ask me. Please do your work on separate paper; you should staple this sheet to your work (put this on top) and turn in everything together.

All questions count equally.

(1) What is a definite integral? Explain thoroughly and draw a picture.

(2) What does the first derivative tell you about the shape of a graph?

(3) (Do Ch. 2.8, 7.)

(4) Find $\frac{dy}{dx}$ if $y = \frac{sin^2 x \tan^4 x}{(x^2+1)^2}$. (Hint: You might want to use logarithmic differentiation.)

(5) Find $\frac{dy}{d\theta}$ if $y = \tan^2(\sin\theta)$.

(6) Evaluate $\int_0^{\pi/3} \frac{\sin\theta + \sin\theta \tan^2\theta}{\sec^2\theta} d\theta$.

(7) Evaluate $\int x^2(x^3+5)^9 dx$.

(8) Graph $f(x) = x + \cos x$. Explicitly describe each of the following: x and y-intercepts; where the graph is positive and negative; critical points; where increasing and decreasing; inflection points; where concave up and concave down; asymptotes if any.

(9) Find the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$.

(10) The region bounded by $y = 1 + \sec x$, $y = 3$, is revolved around the line $y = 1$. Sketch the region, the solid, and a typical slice, and compute the volume of the region.

(11) Is the integral $\int_{-1}^{4} x^2 dx$ defined, and can one use the Fundamental Theorem of Calculus to evaluate it? Why or why not?

(12) Sketch the graph of a function f that is continuous on $[1, 5]$, and has an absolute minimum at 1, an absolute maximum at 5, a local maximum at 2, and a local minimum at 4.

(13) Find the derivative of $g(t) = \frac{1}{\sqrt{t}}$ using the definition of the derivative. (Do **not** just use the power rule, although this will allow you to check that your computation is correct.) State the domain of the function and the domain of its derivative.

(14) A rock is thrown upward on Mars with a velocity of 10 m/s. Its height in meters after t seconds is given by the equation $H = 10t - 1.86t^2$.

Find the velocity of the rock after 1 second. When will the rock hit the surface, and with what velocity?

(15) (Do 5.3, 4.)

(16) A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.