Final Examination (Version 2) - Math 141, Frank Thorne (thornef@mailbox.sc.edu)

Thursday, December 13, 2012

Please work without books, notes, calculators, or any assistance from others. If you have any questions, feel free to ask me. Please do your work on separate paper; you should staple this sheet to your work (put this on top) and turn in everything together.

All questions count equally.

(1) Give the definition of the derivative of a function $f(x)$ at the point $x = a$. Give the algebraic definition, using an equation. Also, draw a picture and explain why your equation gives the slope of the tangent line to the graph of $f(x)$ at $x = a$.

(2) What is the substitution rule for integrals? What does it have to do with the Chain Rule for integrals?

(3) (Do Ch. 2.8, 8.)

(4) Differentiate $g(t) = t^3 \cos t$.

(5) Differentiate $f(x) = \ln(xe^x)$.

(6) Evaluate $\int_0^{pi/4} \frac{1+\cos^2\theta}{\cos^2\theta} d\theta$.

(7) Evaluate $\int \frac{(\ln x)^2}{x} dx$.

(8) Graph $f(x) = e^{-x^2}$. Explicitly describe each of the following: x and y-intercepts; where the graph is positive and negative; critical points; where increasing and decreasing; inflection points; where concave up and concave down; asymptotes if any.

(9) A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together?

(10) If \$1000 is borrowed at 8% interest, find the amounts due at the end of three years if the interest is compounded (a) annually, (b) monthly, (c) hourly, and (d) continously.

(11) The region bounded by $y = \ln x$, $y = 1$, $y = 2$, and $x = 0$ is revolved around the y-axis. Sketch the region, the solid, and a typical slice, and compute the volume of the region.

(12) (Do. 3.7, Problem 6.)

(13) Is the integral $\int_{-1}^{4} x^2 dx$ defined, and can one use the Fundamental Theorem of Calculus to evaluate it? Why or why not?

(14) Use the quotient rule to differentiate the function $f(x) = \frac{\tan x - 1}{\sec x}$.

Then, find $f'(x)$ by first writing $f(x)$ in terms of sines and cosines and then using the rules for differentiation.

Show that your two answers are equivalent.

(15) Sketch the graph of $f(t) = t + \cos t$, for $-2\pi \leq t \leq 2\pi$. Be sure to find the intervals of increase or decrease, the local maximum and minimum values, the intervals of concavity, and the inflection points.

(16) Define the terms *local maximum, local minimum, absolute maximum, absolute minimum*, and *critical number*.