Practice Exam 1 Solutions (141 F12)

1. (a) $\lim_{x \to 1^-} f(x) = 2$ because f approaches 2 for x near 1 and to the left of it.

(b) $\lim_{x \to 1} f(x)$ does not exist because f approaches 2 from the left but 3 from the right.

(c) $\lim_{x \to 5^-} f(x) = 4$ because f approaches 4 when x is near 5 and gets closer to it. The fact that $f(5)$ is not defined is irrelevant.

2.
$$\lim_{x \to 7} \frac{\sqrt{x+2} - 3}{x - 7} = \lim_{x \to 7} \frac{(\sqrt{x+2} - 3)(\sqrt{x+2} + 3)}{(x-7)(\sqrt{x+2} + 3)}$$
$$= \lim_{x \to 7} \frac{(x+2) - 9}{(x-7)(\sqrt{x+2} + 3)}$$
$$= \lim_{x \to 7} \frac{x - 7}{(x-7)(\sqrt{x+2} + 3)}$$
$$= \lim_{x \to 7} \frac{1}{\sqrt{x+2} + 3} = \frac{1}{\sqrt{7+2} + 3}$$
$$= \frac{1}{6}.$$

3.
$$\lim_{x \to -\infty} \frac{1 - x - x^2}{2x^2 - 7} = \lim_{x \to -\infty} \frac{\frac{1}{x^2} - \frac{x}{x^2} - \frac{x^2}{x^2}}{\frac{2x^2}{x^2} - \frac{7}{x^2}}$$
$$= \frac{0 - 0 - 1}{2 - 0} = \frac{-1}{2}.$$

4. $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(4-(x+h)+(x+h)^2) - (4-x+x^2))}{h}$$

$$= \lim_{h \to 0} \frac{(4-x-h+x^2+2xh+h^2) - 4+x-x^2}{h}$$

$$= \lim_{h \to 0} \frac{-h+2xh+h^2}{h} = \lim_{h \to 0} -1+2x+h$$

$$= -1+2x.$$

5. $$G'(x) = \frac{d}{dx}(x^{1/2}) - 2\frac{d}{dx}(e^x)$$

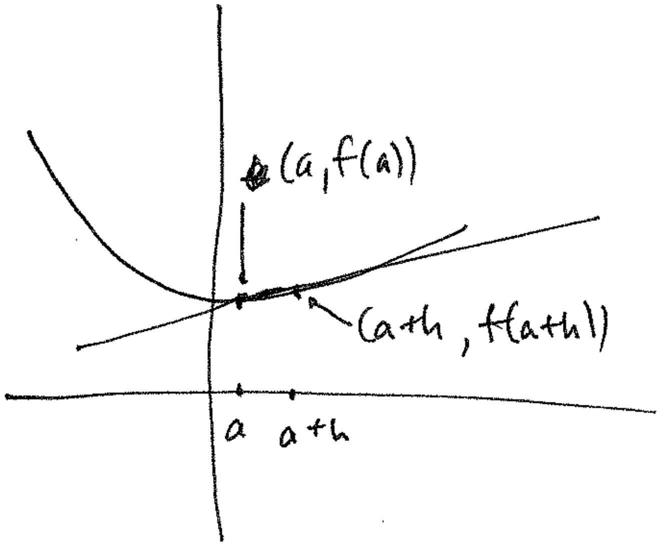
$$= \frac{1}{2}x^{-1/2} - 2e^x.$$

6. $$\frac{dy}{ds} = \frac{(s+ke^s)\frac{d}{ds}(1) - 1\cdot\frac{d}{ds}(s+ke^s)}{(s+ke^s)^2}$$

$$= \frac{(s+ke^s)\cdot 0 - 1\cdot(1+ke^s)}{(s+ke^s)^2}$$

$$= \frac{-1-ke^s}{(s+ke^s)^2}.$$

7. By definition $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.



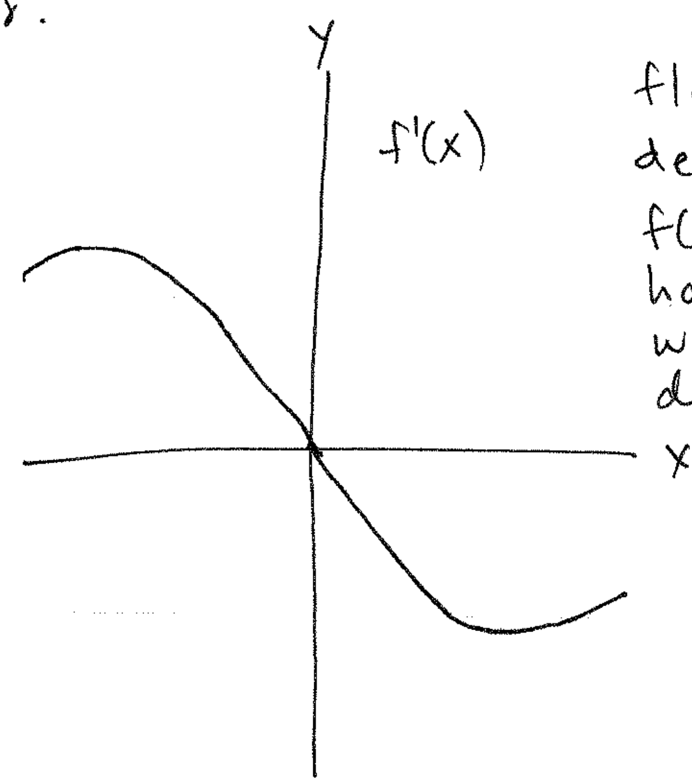
Draw the secant line between $(a, f(a))$ and $(a+h, f(a+h))$. Its slope is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}.$$

As we take the point $a+h$ closer and closer to a, the secant line approaches the tangent line, so

$$\frac{f(a+h) - f(a)}{h} \text{ approaches } \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = f'(a).$$

8.



$f'(x)$ is zero where $f(x)$ is flat, negative where $f(x)$ is decreasing, and positive where $f(x)$ is increasing. $f'(x)$ has its highest and lowest points where $f(x)$ increases and decreases the fastest.