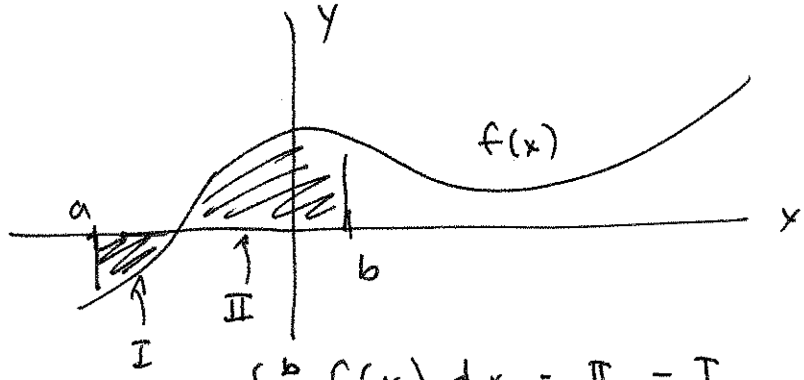
Answers to practice exam. (excluding problems done in class)

1. A definite integral $\int_a^b f(x)\,dx$ represents the area under the graph of $f(x)$ and above the x-axis, between $x = a$ and $x = b$. When $f(x) < 0$ it is counted negative.

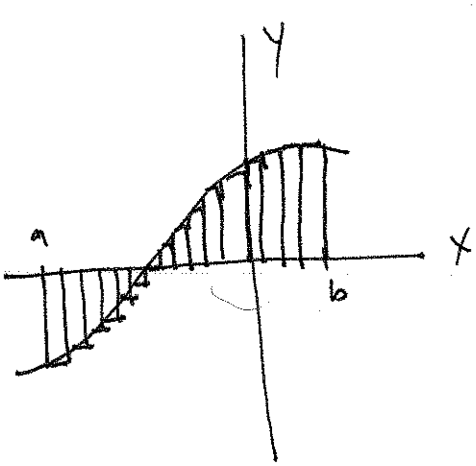


$$\int_a^b f(x)\,dx = \mathrm{II} - \mathrm{I}.$$

A formula is

$$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \Big(f(a)\,\Delta x + f(a+\Delta x)\,\Delta x + f(a+2\Delta x)\,\Delta x + \cdots + f(b-\Delta x)\,\Delta x \Big).$$

It represents chopping the region into very narrow rectangles and adding the areas, and taking a limit as $\Delta x \to 0$.



← Above quantity in parentheses is the (signed) area of the rectangles, if each rectangle has width Δx.

4. $\int_0^{\pi/4} \sec^2 t \, dt.$ Recall $\frac{d}{dt}(\tan t) = \sec^2 t.$

So this is $\left[\tan t\right]_0^{\pi/4} = \tan(\pi/4) - \tan(0)$

$= 1 - 0 = 1.$

6. $\int_0^7 \sqrt{4+3x}\, dx.$ Set ~~$u = \sqrt{4+3x}$~~.

$u = 4 + 3x,$

Then $\frac{du}{dx} = 3$, so $du = 3dx.$

and, $dx = \frac{du}{3}.$

$$\int_0^7 \sqrt{4+3x}\, dx = \int_{x=0}^{x=7} \sqrt{u} \cdot \frac{du}{3}$$

If $u = 4+3x$,

$x = 0 \Rightarrow u = 4$

$x = 7 \Rightarrow u = 25,$

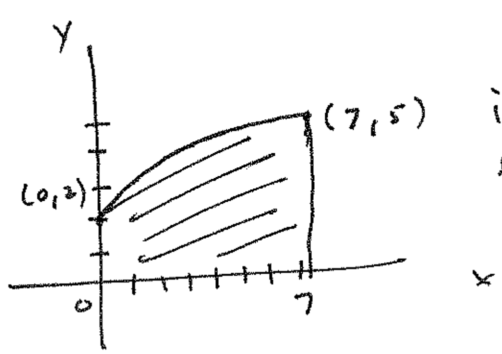
$$= \int_{u=4}^{u=25} u^{1/2} \cdot \frac{du}{3}$$

$$= \left[\frac{u^{3/2}}{3/2} \cdot \frac{1}{3}\right]_4^{25}$$

$$= \left[\frac{2}{9} u^{3/2}\right]_4^{25} = \frac{2}{9} \cdot 125 - \frac{2}{9} \cdot 8$$

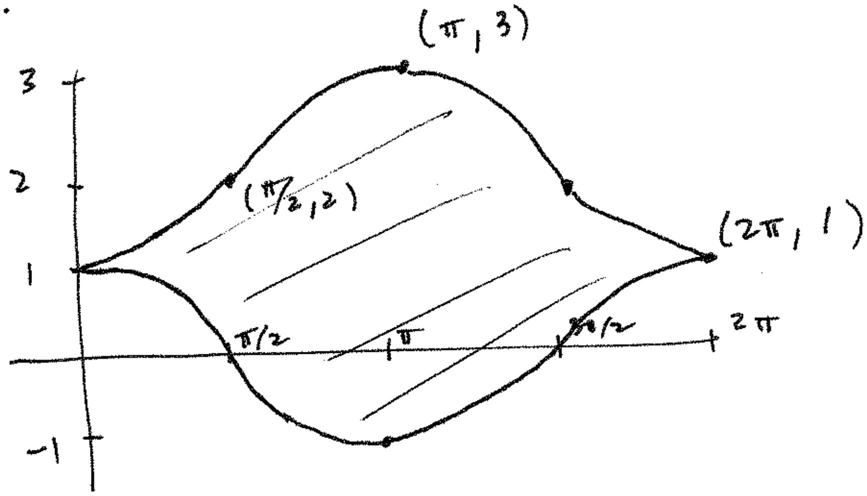
$$= \frac{2}{9} \cdot 117 = 2 \cdot 13 = 26.$$

This integral represents this area:



Note. We can tell the area is between $2 \cdot 7 = 14$ and $5 \cdot 7 = 35$ from the picture.

7.



The area is $\int_0^{2\pi} \left((2-\cos x) - \cos x \right) dx$

$$= \int_0^{2\pi} (2 - 2\cos x)\, dx$$

$$= \Big[2x - 2\sin x \Big]_0^{2\pi}$$

$$= \left(2 \cdot (2\pi) - 2\sin(2\pi)\right) - \left(2 \cdot 0 - 2\sin(0)\right)$$

$$= 4\pi - 0 - (0 - 0) = 4\pi .$$